

Surface Area

What You'll Learn

- **Lesson 12-1** Identify three-dimensional figures.
- **Lesson 12-2** Draw two-dimensional models for solids.
- **Lessons 12-3 through 12-6** Find the lateral areas and surface areas of prisms, cylinders, pyramids, and cones.
- **Lesson 12-7** Find the surface areas of spheres and hemispheres.

Key Vocabulary

- polyhedron (p. 637)
- net (p. 644)
- surface area (p. 644)
- lateral area (p. 649)

Why It's Important

Diamonds and other gems are cut to enhance the beauty of the stones. The stones are cut into regular geometric shapes. Each cut has a special name.

You will learn more about gemology in Lesson 12-1.

Getting Started

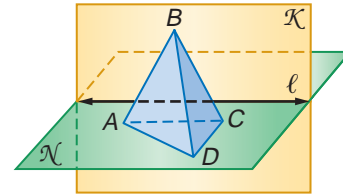
Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 12.

For Lesson 12-1

Parallel Lines and Planes

In the figure, $\overline{AC} \parallel \ell$. Determine whether each statement is *true*, *false*, or *cannot be determined*. (For review, see Lesson 3-1.)

- $\triangle ADC$ lies in plane \mathcal{K} .
- $\triangle ABC$ lies in plane \mathcal{K} .
- The line containing \overline{AB} is parallel to plane \mathcal{K} .
- The line containing \overline{AC} lies in plane \mathcal{K} .

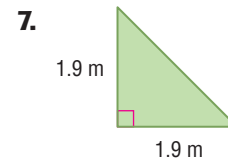
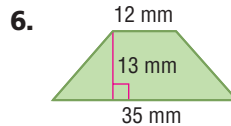
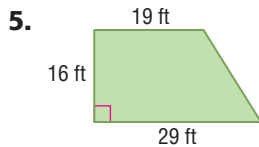


For Lessons 12-3 and 12-5

Areas of Triangles and Trapezoids

Find the area of each figure. Round to the nearest tenth if necessary.

(For review, see Lesson 11-2.)



For Lessons 12-4, 12-6, and 12-7

Area of Circles

Find the area of each circle with the given radius or diameter. Round to the nearest tenth.

(For review, see Lesson 11-3.)

8. $d = 19.0$ cm

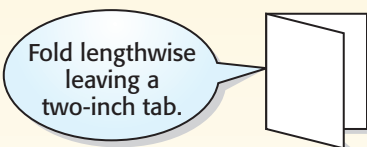
9. $r = 1.5$ yd

10. $d = 10.4$ m

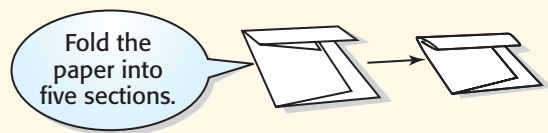
FOLDABLES™ Study Organizer

Surface Area Make this Foldable to help you organize what you learn about surface area. Begin with a large sheet of paper, such as 11 inches \times 17 inches.

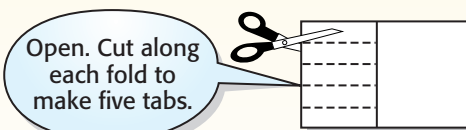
Step 1 Fold Lengthwise



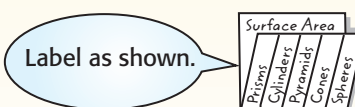
Step 2 Fold



Step 3 Cut



Step 4 Label



Reading and Writing As you read and study the chapter, define terms and write notes about surface area for each three-dimensional figure.

What You'll Learn

- Use orthogonal drawings of three-dimensional figures to make models.
- Identify and use three-dimensional figures.

Vocabulary

- orthogonal drawing
- corner view
- perspective view
- polyhedron
- face
- edges
- prism
- bases
- regular prism
- pyramid
- regular polyhedron
- Platonic solids
- cylinder
- cone
- sphere
- cross section

Why

are drawings of three-dimensional structures valuable to archaeologists?

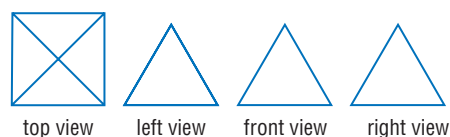
Archaeologists and Egyptologists continue to study the Great Pyramids of Egypt. Drawings of these three-dimensional structures are helpful in their study.



DRAWINGS OF THREE-DIMENSIONAL FIGURES

If you see a three-dimensional object from only one viewpoint, you may not know its true shape.

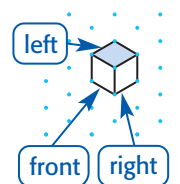
Here are four views of the pyramid of Menkaure in Giza, Egypt. The two-dimensional views of the top, left, front, and right sides of an object are called an **orthogonal drawing**.



This sculpture is *Stacked Pyramid* by Jackie Ferrara. How can we show the stacks of blocks on each side of the piece in a two-dimensional drawing? Let the edge of each block represent a unit of length and use a dark segment to indicate a break in the surface.



The view of a figure from a corner is called the **corner view** or **perspective view**. You can use isometric dot paper to draw the corner view of a solid figure. One corner view of a cube is shown at the right.



Study Tip

Corner View Drawings

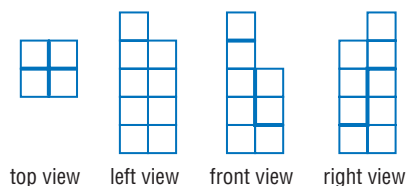
Use models to help you visualize the corner view of a solid.

Example 1 Use Orthogonal Drawings

- a. Draw the back view of a figure given its orthogonal drawing.

Use blocks to make a model. Then use your model to draw the back view.

- The top view indicates two rows and two columns of different heights.
- The front view indicates that the left side is 5 blocks high and the right side is 3 blocks high. The dark segments indicate breaks in the surface.



- The right view indicates that the right front column is only one block high. The left front column is 4 blocks high. The right back column is 3 blocks high.
- Check the left side of your model. All of the blocks should be flush.
- Check to see that all views correspond to the model.



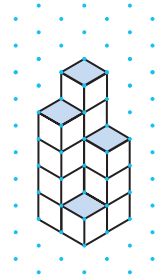
Now that your model is accurate, turn it around to the back and draw what you see. The blocks are flush, so no heavy segments are needed.



b. Draw the corner view of the figure.

Turn your model so you are looking at the corners of the blocks. The lowest columns should be in front so the differences in height between the columns is visible.

Connect the dots on the isometric dot paper to represent the edges of the solid. Shade the tops of each column.



Study Tip

Reading Math

The plural version of a polyhedron is either *polyhedrons* or *polyhedra*. Both forms are correct.

IDENTIFY THREE-DIMENSIONAL FIGURES

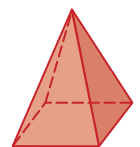
A solid with all flat surfaces that enclose a single region of space is called a **polyhedron**. Each flat surface, or **face**, is a polygon. The line segments where the faces intersect are called **edges**. Edges intersect at a point called a *vertex*.

A **prism** is a polyhedron with two parallel congruent faces called **bases**. The other faces are parallelograms. The intersection of three edges is a vertex. Prisms are named by the shape of their bases. A **regular prism** is a prism with bases that are regular polygons. A cube is an example of a regular prism. Some common prisms are shown below.

Name	Triangular Prism	Rectangular Prism	Pentagonal Prism
Model			
Shape of Base	triangle	rectangle	pentagon

A polyhedron with all faces (except for one) intersecting at one vertex is a **pyramid**. Pyramids are named for their bases, which can be any polygon.

A polyhedron is a **regular polyhedron** if all of its faces are regular congruent polygons and all of the edges are congruent.



square pyramid

There are exactly five types of regular polyhedra. These are called the **Platonic Solids** because Plato described them extensively in his writings.

More About . . .

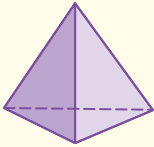
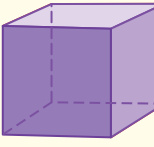
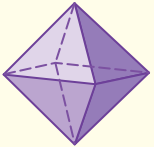
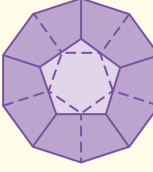
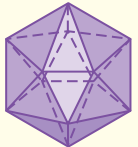


Plato

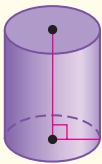
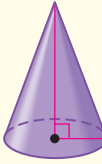
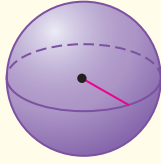
Plato was a teacher of mathematics and philosophy. Around 387 B.C., he founded a school in Athens, Greece, called the "Academy."

Source: www.infoplease.com



Platonic Solids					
Name	tetrahedron	hexahedron	octahedron	dodecahedron	icosahedron
Model					
Number of Faces	4	6	8	12	20
Shape of Face	equilateral triangle	square	equilateral triangle	regular pentagon	equilateral triangle

There are solids that are *not* polyhedrons. All of the faces in each solid are not polygons. A **cylinder** is a solid with congruent circular bases in a pair of parallel planes. A **cone** has a circular base and a vertex. A **sphere** is a set of points in space that are a given distance from a given point.

Name	cylinder	cone	sphere
Model			

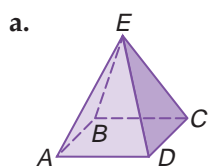
Study Tip

Common Misconception

Prisms can be oriented so the bases are not the top and bottom of the solid.

Example 2 Identify Solids

Identify each solid. Name the bases, faces, edges, and vertices.



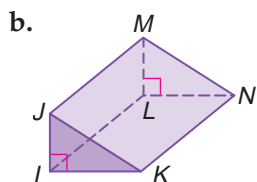
The base is a rectangle, and the other four faces meet in a point. So this solid is a rectangular pyramid.

Base: $\square ABCD$

Faces: $\square ABCD$, $\triangle AED$, $\triangle DEC$, $\triangle CEB$, $\triangle AEB$

Edges: \overline{AB} , \overline{BC} , \overline{CD} , \overline{DA} , \overline{AE} , \overline{DE} , \overline{CE} , \overline{BE}

Vertices: A, B, C, D, E



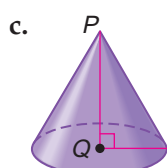
The bases are right triangles. So this is a triangular prism.

Bases: $\triangle IJK$, $\triangle LMN$

Faces: $\triangle IJK$, $\triangle LMN$, $\square ILNK$, $\square KJMN$, $\square IJML$

Edges: \overline{IL} , \overline{LN} , \overline{NK} , \overline{IK} , \overline{IJ} , \overline{LM} , \overline{JM} , \overline{MN} , \overline{JK}

Vertices: I, J, K, L, M, N



This solid has a circle for a base and a vertex. So it is a cone.

Base: $\odot Q$

Vertex: P

Interesting shapes occur when a plane intersects, or slices, a solid figure. If the plane is parallel to the base or bases of the solid, then the intersection of the plane and solid is called a **cross section** of the solid.

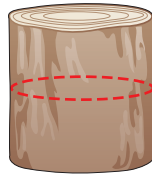
Study Tip

Modeling

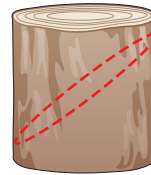
You can model each of these cuts by using modeling clay for the cylinder and cutting the clay with dental floss.

Example 3 Slicing Three-Dimensional Figures

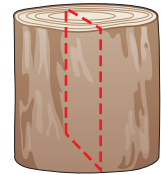
CARPENTRY A carpenter purchased a section of a tree trunk. He wants to cut the trunk into a circle, an oval, and a rectangle. How could he cut the tree trunk to get each shape?



The tree trunk has a cylindrical shape. If the blade of the saw was placed parallel to the bases, the cross section would be a circle.



If the blade was placed at an angle to the bases of the tree trunk, the slice would be an oval shape, or an ellipse.

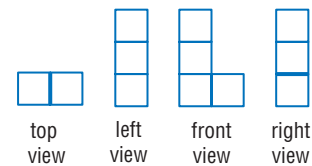


To cut a rectangle from the cylinder, place the blade perpendicular to the bases. The slice is a rectangle.

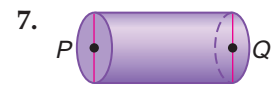
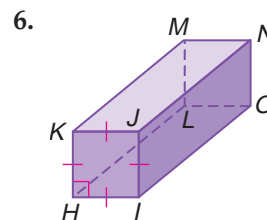
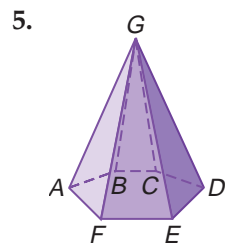
Check for Understanding

- Concept Check**
1. Explain how the Platonic Solids are different from other polyhedra.
 2. Explain the difference between a square pyramid and a square prism.
 3. **OPEN ENDED** Draw a rectangular prism.

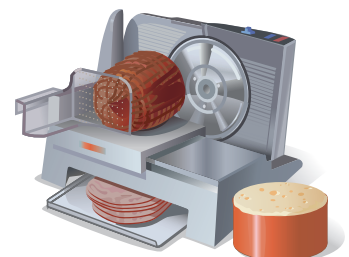
- Guided Practice**
4. Draw the back view and corner view of a figure given its orthogonal drawing.



Identify each solid. Name the bases, faces, edges, and vertices.



- Application**
8. **DELICATESSEN** A slicer is used to cut whole pieces of meat and cheese for sandwiches. Suppose a customer wants slices of cheese that are round and slices that are rectangular. How can the cheese be placed on the slicer to get each shape?



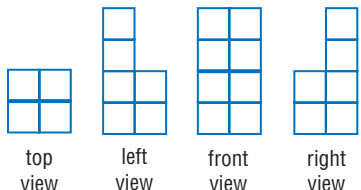
Practice and Apply

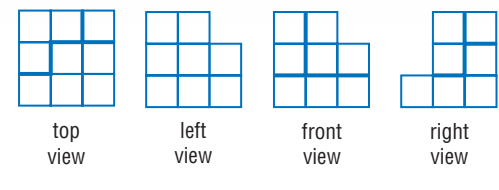
Homework Help

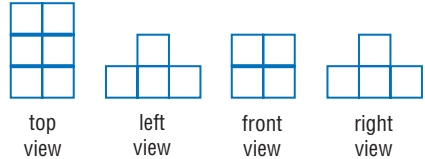
For Exercises	See Examples
9–15, 23, 24	1
16–22, 36–41	2
25–35	3

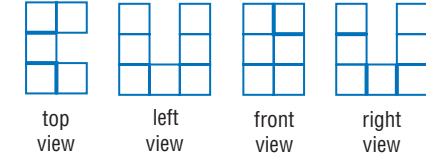
Extra Practice
See page 778.

Draw the back view and corner view of a figure given each orthogonal drawing.


9.  top view left view front view right view

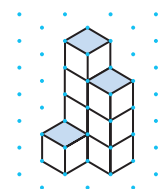
10.  top view left view front view right view


11.  top view left view front view right view

12.  top view left view front view right view

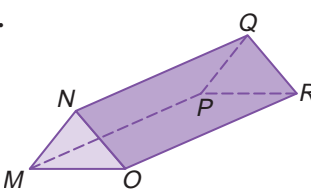
Given the corner view of a figure, sketch the orthogonal drawing.

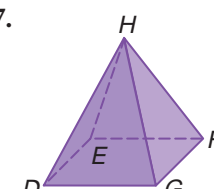
13. 

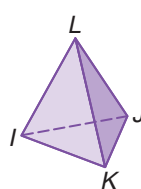
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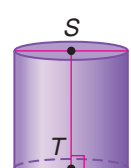
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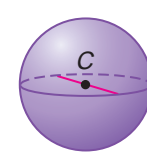
Identify each solid. Name the bases, faces, edges, and vertices.

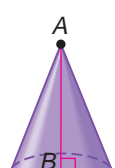
16. 

17. 

18. 

19. 

20. 

21. 

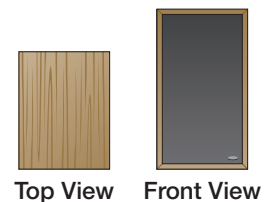
22. **EULER'S FORMULA** The number of faces F , vertices V , and edges E of a polyhedron are related by Euler's (OY luhrz) formula: $F + V = E + 2$. Determine whether Euler's formula is true for each of the figures in Exercises 16–21.

SPEAKERS For Exercises 23 and 24, use the following information.

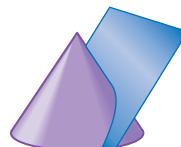
The top and front views of a speaker for a stereo system are shown.

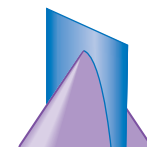
23. Is it possible to determine the shape of the speaker? Explain.

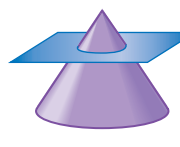
24. Describe possible shapes for the speaker. Draw the left and right views of one of the possible shapes.



Determine the shape resulting from each slice of the cone.

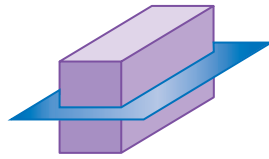
25. 

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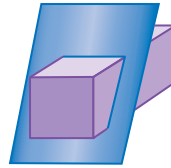
27. 

Determine the shape resulting from each slice of the rectangular prism.

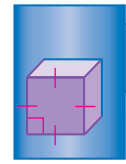
28.



29.



30.



Draw a diagram and describe how a plane can slice a tetrahedron to form the following shapes.

31. equilateral triangle

32. isosceles triangle

33. quadrilateral

Describe the solid that results if the number of sides of each base increases infinitely.

34. pyramid

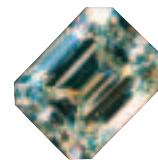
35. prism

GEMOLOGY For Exercises 36–38, use the following information.

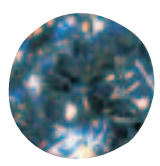
A well-cut diamond enhances the natural beauty of the stone. These cuts are called *facets*.



uncut



emerald cut



round cut

36. Describe the shapes seen in an uncut diamond.

37. What shapes are seen in the emerald-cut diamond?

38. List the shapes seen in the round-cut diamond.

For Exercises 39–41, use the following table.

Number of Faces	Prism	Pyramid
4	none	tetrahedron
5	a. _____ ? _____	square or rectangular
6	b. _____ ? _____	c. _____ ? _____
7	pentagonal	d. _____ ? _____
8	e. _____ ? _____	heptagonal

39. Name the type of prism or pyramid that has the given number of faces.

40. Analyze the information in the table. Is there a pattern between the number of faces and the bases of the corresponding prisms and pyramids?

41. Is it possible to classify a polyhedron given only the number of faces? Explain.

42. **CRITICAL THINKING** Construct a Venn diagram that shows the relationship among polyhedrons, Platonic solids, prisms, and pyramids.

43. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why are drawings of three-dimensional structures valuable to archaeologists?

Include the following in your answer:

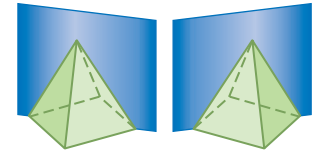
- types of two-dimensional models and drawings, and
- the views of a structure used to show three dimensions.



44. All of the following can be formed by the intersection of a cube and a plane *except*
 (A) a triangle. (B) a rectangle. (C) a point. (D) a circle.
45. **ALGEBRA** For which of the following values of x is $\frac{x^3}{x^4}$ the least?
 (A) -4 (B) -3 (C) -2 (D) -1

**Extending
the Lesson**

SYMMETRY AND SOLIDS In a two-dimensional plane, figures are symmetric with respect to a line or a point. In three-dimensional space, solids are symmetric with respect to a plane. A square pyramid has four planes of symmetry. Two pass through the height and each vertex of the base, and two pass through the height and the center of each edge of the base.



For each solid, determine the number of planes of symmetry.

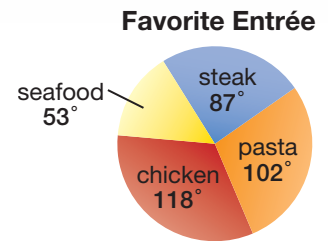
46. tetrahedron 47. cylinder 48. sphere

Maintain Your Skills

Mixed Review

SURVEYS For Exercises 49–52, use the following information.

The results of a restaurant survey are shown in the circle graph with the measurement of each central angle. Each customer was asked to choose a favorite entrée. If a customer is chosen at random, find the probability of each response. (Lesson 11-5)



49. steak 50. not seafood
 51. either pasta or chicken 52. neither pasta nor steak

COORDINATE GEOMETRY The coordinates of the vertices of an irregular figure are given. Find the area of each figure. (Lesson 11-4)

53. $A(1, 4), B(4, 1), C(1, -2), D(-3, 1)$
 54. $F(-2, -4), G(-2, -1), H(1, 1), J(4, 1), K(6, -4)$
 55. $L(-2, 2), M(0, 1), N(0, -2), P(-4, -2)$

Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary. (Lesson 11-1)

56. 57. 58.

**Getting Ready for
the Next Lesson**

PREREQUISITE SKILL Find the area of each rectangle. Round to the nearest tenth if necessary. (To review finding the area of a rectangle, see pages 732–733.)

59. 60. 61. 62.

12-2

Nets and Surface Area

What You'll Learn

- Draw two-dimensional models for three-dimensional figures.
- Find surface area.

Vocabulary

- net
- surface area

Why is surface area important to car manufacturers?

Have you wondered why cars have evolved from boxy shapes to sleeker shapes with rounded edges? Car manufacturers use aerodynamics, or the study of wind resistance, and the shapes of surfaces to design cars that are faster and more efficient.



Study Tip

Isometric Dot Paper

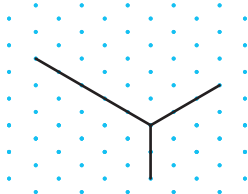
Note that right angles of the prism are 60° and 120° angles on isometric dot paper. This is to show perspective.

MODELS FOR THREE-DIMENSIONAL FIGURES You have used isometric dot paper to draw corner views of solids given the orthogonal view. In this lesson, isometric dot paper will be used to draw two-dimensional models of geometric solids.

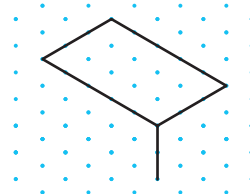
Example 1 Draw a Solid

Sketch a rectangular prism 2 units high, 5 units long, and 3 units wide using isometric dot paper.

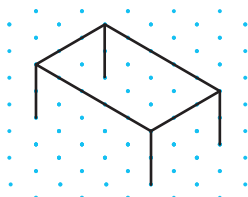
Step 1 Draw the corner of the solid; 2 units down, 5 units to the left, and 3 units to the right.



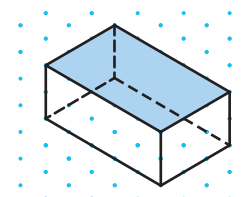
Step 2 Draw a parallelogram for the top of the solid.



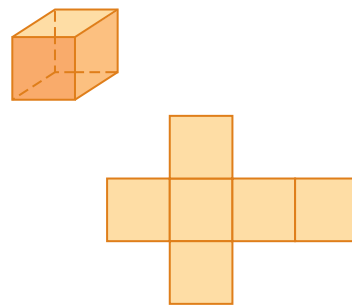
Step 3 Draw segments 2 units down from each vertex for the vertical edges.



Step 4 Connect the corresponding vertices. Use dashed lines for the hidden edges. Shade the top of the solid.



If you cut a cardboard box at the edges and lay it flat, you will have a pattern, or **net**, for the three-dimensional solid. Nets can be made for any solid figure. This net is a pattern for the cube. It can be folded into the shape of the cube without any overlap.



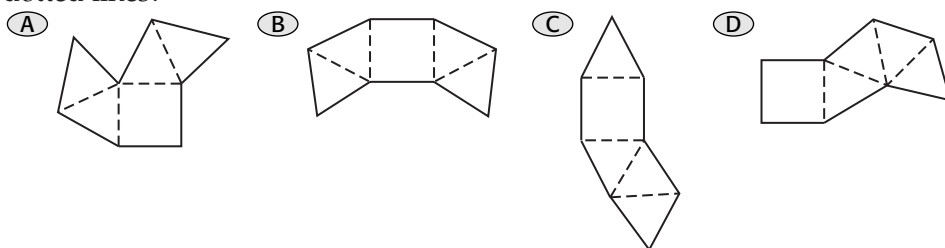
Standardized Test Practice

A B C D

Example 2 Nets for a Solid

Multiple-Choice Test Item

Which net could be folded into a pyramid if folds are made only along the dotted lines?



The Princeton Review

Test-Taking Tip

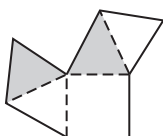
Nets One figure may have several different nets that represent the shape.

Read the Test Item

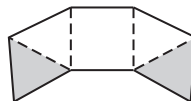
You are given four nets, only one of which can be folded into a pyramid.

Solve the Test Item

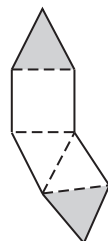
Each of the answer choices has one square and four triangles. So the square is the base of the pyramid. Each triangle in the sketch represents a face of the pyramid. The faces must meet at a point and cannot overlap. Analyze each answer choice carefully.



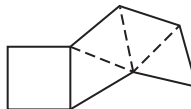
This net has overlapping triangles.



This net also has two triangles that overlap.



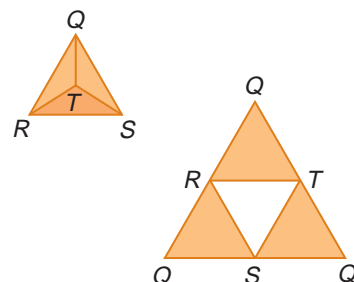
This also has overlapping triangles.



None of the triangles overlap. Each face of the pyramid is represented. This choice is correct.

The answer is D.

SURFACE AREA Nets are very useful in visualizing the polygons that make up the surface of the solid. A net for tetrahedron $QRST$ is shown at the right. The **surface area** is the sum of the areas of each face of the solid. Add the areas of $\triangle QRT$, $\triangle QTS$, $\triangle QRS$, and $\triangle RST$ to find the surface area of tetrahedron $QRST$.



Study Tip

Drawing Nets

It is helpful to let each unit of the dot paper represent one unit of measure. When the numbers are large, let each unit of dot paper represent two units of measure.

Example 3 Nets and Surface Area

- a. Draw a net for the right triangular prism shown.

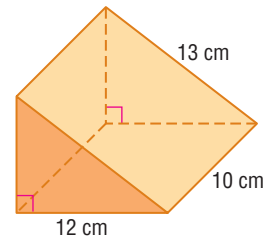
We need to know the height of the prism. Use the Pythagorean Theorem to find the height of the prism.

$$13^2 = 12^2 + h^2 \quad \text{Pythagorean Theorem}$$

$$169 = 144 + h^2 \quad \text{Simplify.}$$

$$25 = h^2 \quad \text{Subtract 144 from each side.}$$

$$5 = h \quad \text{Take the square root of each side.}$$



Use rectangular dot paper to draw a net. Let one unit on the dot paper represent 2 centimeters.

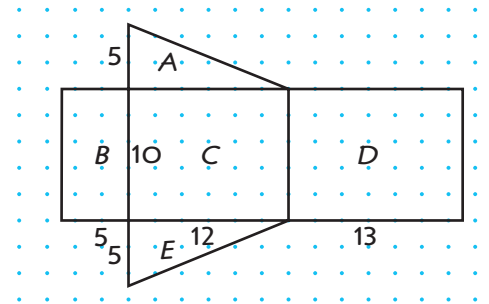
- b. Use the net to find the surface area of the triangular prism.

To find the surface area of the prism, add the areas of the three rectangles and the two triangles.

Write an equation to solve for the surface area.

$$\begin{aligned} \text{Surface area} &= B + C + D + A + E \\ &= 10 \cdot 5 + 10 \cdot 12 + 10 \cdot 13 + \frac{5 \cdot 12}{2} + \frac{5 \cdot 12}{2} \\ &= 50 + 120 + 130 + 30 + 30 \text{ or } 360 \end{aligned}$$

The surface area of the right triangular prism is 360 square centimeters.



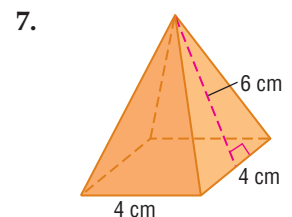
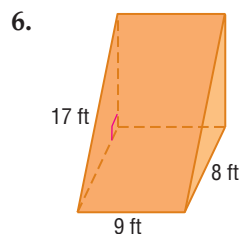
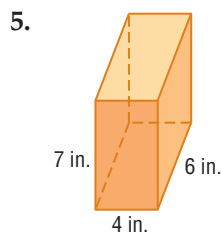
Check for Understanding

- Concept Check**
- OPEN ENDED** Draw a net for a cube different from the one on page 644.
 - Compare and contrast** isometric dot paper and rectangular dot paper. When is each type of paper useful?

Guided Practice Sketch each solid using isometric dot paper.

- rectangular prism 4 units high, 2 units long, and 3 units wide
- cube 2 units on each edge

For each solid, draw a net and find the surface area.



8. Which shape *cannot* be folded to make a pyramid?

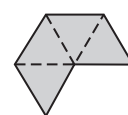
(A)



(B)



(C)



(D)



Standardized Test Practice

(A) (B) (C) (D)

Practice and Apply

Homework Help

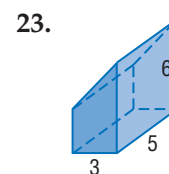
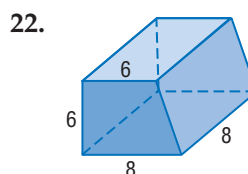
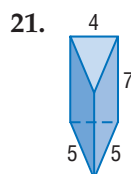
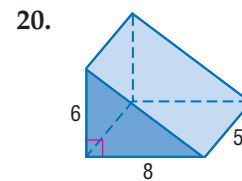
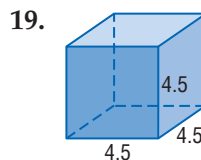
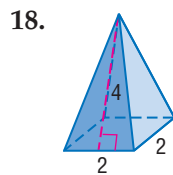
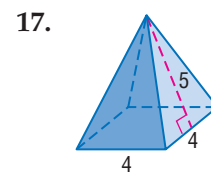
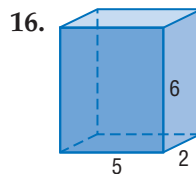
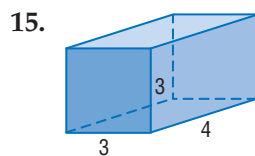
For Exercises	See Examples
9–14, 25–27	1
15–24, 35–38	2
28–34	3

Extra Practice
See page 778.

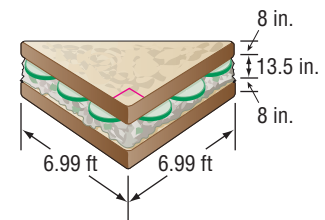
Sketch each solid using isometric dot paper.

- rectangular prism 3 units high, 4 units long, and 5 units wide
- cube 5 units on each edge
- cube 4 units on edge
- rectangular prism 6 units high, 6 units long, and 3 units wide
- triangular prism 4 units high, with bases that are right triangles with legs 5 units and 4 units long
- triangular prism 2 units high, with bases that are right triangles with legs 3 units and 7 units long

For each solid, draw a net and find the surface area. Round to the nearest tenth if necessary.

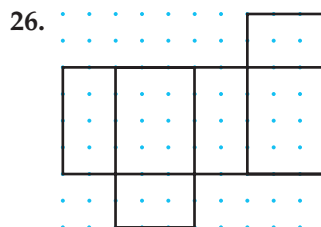
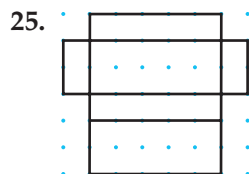


24. **FOOD** In 1999, Marks & Spencer, a British grocery store, created the biggest sandwich ever made. The tuna and cucumber sandwich was in the form of a triangular prism. Suppose each slice of bread was 8 inches thick. Draw a net of the sandwich, and find the surface area in square feet to the nearest tenth.



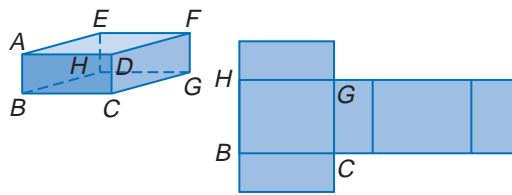
Online Research Data Update Are there records for other types of sandwiches? Visit www.geometryonline.com/data_update to learn more.

Given the net of a solid, use isometric dot paper to draw the solid.

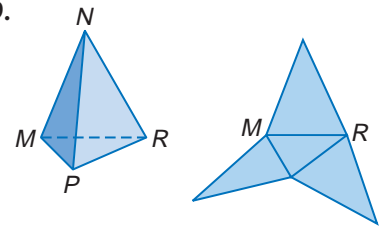


Given each polyhedron, copy its net and label the remaining vertices.

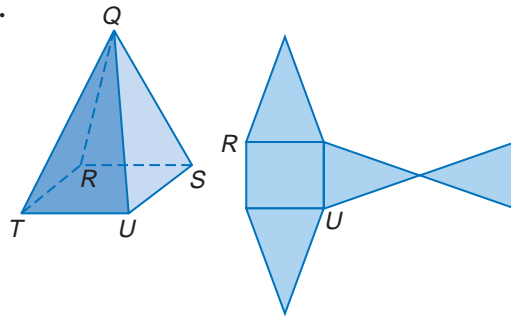
28.



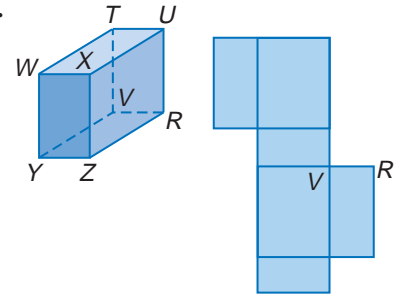
29.



30.



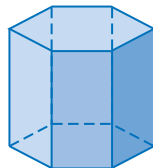
31.



• GEOLOGY For Exercises 32–34, use the following information.

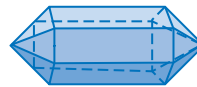
Many minerals have a crystalline structure. The forms of three minerals are shown below. Draw a net of each crystal.

32.



tourmaline

33.



quartz

34.



calcite

VARYING DIMENSIONS For Exercises 35–38, use Figures A, B, and C.

35. Draw a net for each solid and find its surface area.

Figure A

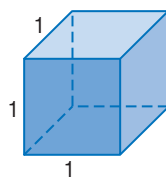


Figure B

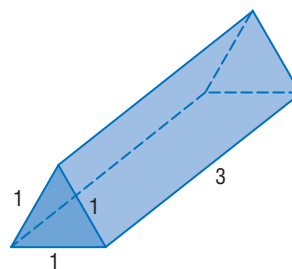
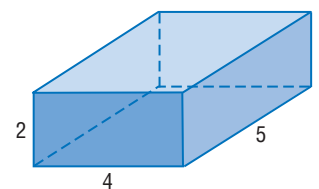


Figure C

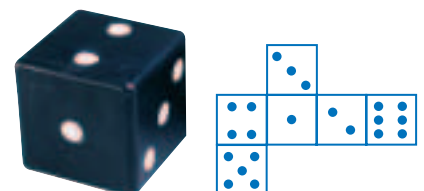


36. Double the dimensions of each figure. Find the surface areas.

37. How does the surface area change when the dimensions are doubled? Explain.

38. Make a conjecture about the surface area of a solid whose dimensions have been tripled. Check your conjecture by finding the surface area.

39. **CRITICAL THINKING** Many board games use a standard die like the one shown. The sum of the number of dots on each pair of opposite faces is 7. Determine whether the net represents a standard die. Explain.



Geology

There are 32 different classes of crystals, each with a different type of symmetry.

Source: www.infoplease.com

40. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

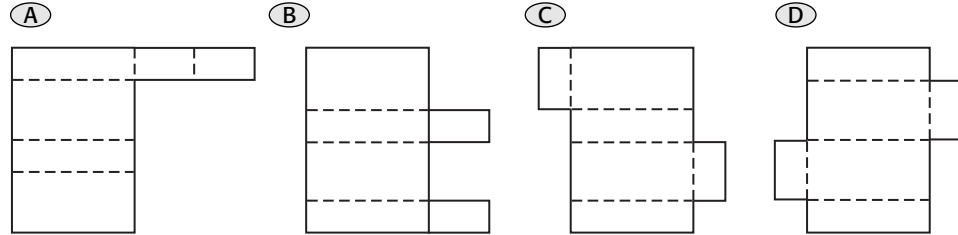
Why is surface area important to car manufacturers?

Include the following in your answer:

- compare the surface area of a subcompact car and a large truck, and
- explain which two-dimensional models of cars would be helpful to designers.



41. Which shape could be folded into a rectangular prism if folds are made only along the dotted lines?



42. **ALGEBRA** What is the complete factorization of $16a^3 - 54b^3$?
- (A) $(2a - 3b)(4a^2 + 6ab + 9b^2)$ (B) $2(2a - 3b)(4a^2 + 6ab + 9b^2)$
 (C) $2(2a - 3b)(4a^2 - 6ab + 9b^2)$ (D) $2(2a + 3b)(4a^2 + 6ab + 9b^2)$

Maintain Your Skills

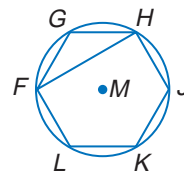
Mixed Review Determine the shape resulting from each slice of the triangular prism.
(Lesson 12-1)



46. **PROBABILITY** A rectangular garden is 100 feet long and 200 feet wide and includes a square flower bed that is 20 feet on each side. Find the probability that a butterfly in the garden is somewhere in the flower bed. *(Lesson 11-5)*

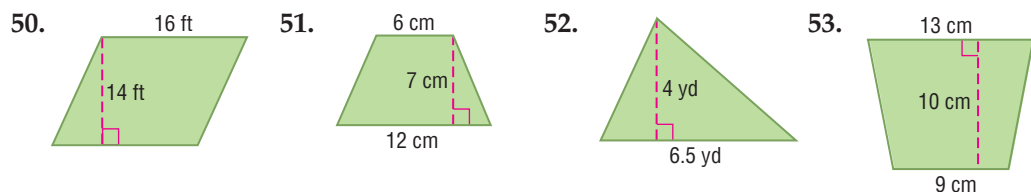
Equilateral hexagon $FGHJKL$ is inscribed in $\odot M$. Find each measure. *(Lesson 10-4)*

47. $m\angle FHJ$
 48. $m\widehat{LK}$
 49. $m\angle LFG$



Getting Ready for the Next Lesson

PREREQUISITE SKILL Find the area of each figure.
(To review finding areas of parallelograms, triangles, and trapezoids, see Lessons 11-1 and 11-2.)



12-3

Surface Areas of Prisms

What You'll Learn

- Find lateral areas of prisms.
- Find surface areas of prisms.

Vocabulary

- lateral faces
- lateral edges
- right prism
- oblique prism
- lateral area

How

do brick masons know how many bricks to order for a project?

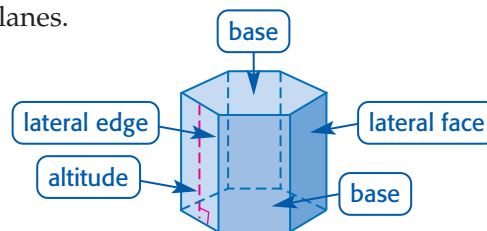
The owner of a house wants to build a new unattached brick garage. The sides of the garage will be brick. The brick mason has to estimate the number of bricks needed to complete the project.



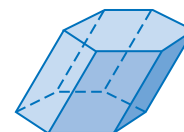
LATERAL AREAS OF PRISMS

Most buildings are prisms or combinations of prisms. The garage shown above could be separated into a rectangular prism and a rectangular pyramid. Prisms have the following characteristics.

- The bases are congruent faces in parallel planes.
- The faces that are not bases are called **lateral faces**.
- The lateral faces intersect at the **lateral edges**. Lateral edges are parallel segments.
- A segment perpendicular to the bases, with an endpoint in each plane, is called an *altitude* of the prism. The height of a prism is the length of the altitude.
- A prism with lateral edges that are also altitudes is called a **right prism**. If the lateral edges are not perpendicular to the bases, it is an **oblique prism**.



right hexagonal prism



oblique hexagonal prism

Study Tip

Lateral Faces

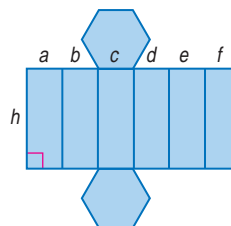
The lateral faces of right prisms are always rectangles.

Study Tip

Reading Math

From this point in the text, you can assume that solids are right solids. If a solid is oblique, it will be clearly stated.

The **lateral area** L is the sum of the areas of the lateral faces.



$$\begin{aligned} L &= ah + bh + ch + dh + eh + fh \\ &= h(a + b + c + d + e + f) \\ &= Ph \end{aligned}$$

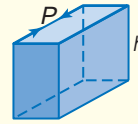
Distributive Property

$$P = a + b + c + d + e + f$$

Key Concept

Lateral Area of a Prism

If a right prism has a lateral area of L square units, a height of h units, and each base has a perimeter of P units, then $L = Ph$.

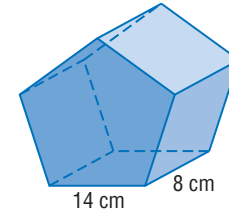


Example 1 Lateral Area of a Pentagonal Prism

Find the lateral area of the regular pentagonal prism.

The bases are regular pentagons. So the perimeter of one base is $5(14)$ or 70 centimeters.

$$\begin{aligned}L &= Ph && \text{Lateral area of a prism} \\ &= (70)(8) && P = 70, h = 8 \\ &= 560 && \text{Multiply.}\end{aligned}$$



The lateral area is 560 square centimeters.

Study Tip

Right Prisms

The bases of a right prism are congruent, but the faces are not always congruent.

SURFACE AREAS OF PRISMS The surface area of a prism is the lateral area plus the areas of the bases. The bases are congruent, so the areas are equal.

Key Concept

Surface Area of a Prism

If the surface area of a right prism is T square units, its height is h units, and each base has an area of B square units and a perimeter of P units, then $T = L + 2B$.

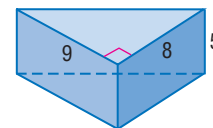


Example 2 Surface Area of a Triangular Prism

Find the surface area of the triangular prism.

First, find the measure of the third side of the triangular base.

$$\begin{aligned}c^2 &= a^2 + b^2 && \text{Pythagorean Theorem} \\ c^2 &= 8^2 + 9^2 && \text{Substitution} \\ c^2 &= 145 && \text{Simplify.} \\ c &= \sqrt{145} && \text{Take the square root of each side.}\end{aligned}$$



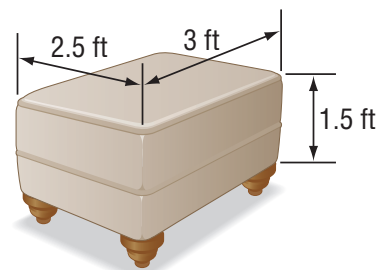
$$\begin{aligned}T &= L + 2B && \text{Surface area of a prism} \\ &= Ph + 2B && L = Ph \\ &= (8 + 9 + \sqrt{145})5 + 2\left[\frac{1}{2}(8 \cdot 9)\right] && \text{Substitution} \\ &\approx 217.2 && \text{Use a calculator.}\end{aligned}$$

The surface area is approximately 217.2 square units.

Example 3 Use Surface Area to Solve a Problem

FURNITURE Rick wants to have an ottoman reupholstered. Find the surface area that will be reupholstered.

The ottoman is shaped like a rectangular prism. Since the bottom of the ottoman is not covered with fabric, find the lateral area and then add the area of one base. The perimeter of a base is $2(3) + 2(2.5)$ or 11 feet. The area of a base is $3(2.5)$ or 7.5 square feet.



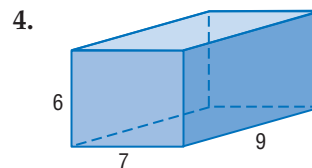
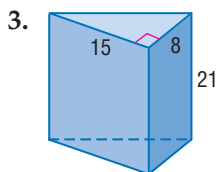
$$\begin{aligned} T &= L + B && \text{Formula for surface area} \\ &= (11)(1.5) + 7.5 && P = 11, h = 1.5, \text{ and } B = 7.5 \\ &= 24 && \text{Simplify.} \end{aligned}$$

The total area that will be reupholstered is 24 square feet.

Check for Understanding

- Concept Check**
1. Explain the difference between a right prism and an oblique prism.
 2. **OPEN ENDED** Draw a prism and label the bases, lateral faces, and lateral edges.

Guided Practice Find the lateral area and surface area of each prism.



- Application**
5. **PAINTING** Eva and Casey are planning to paint the walls and ceiling of their living room. The room is 20 feet long, 15 feet wide, and 12 feet high. Find the surface area to be painted.

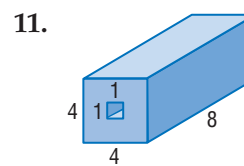
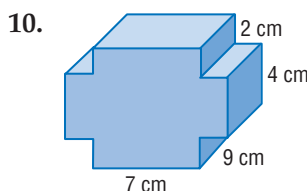
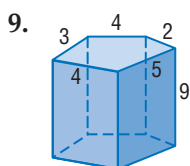
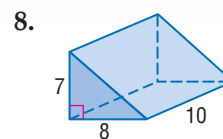
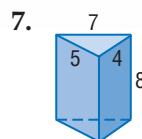
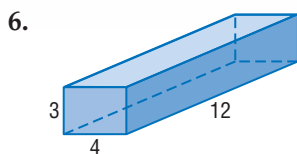
Practice and Apply

Homework Help

For Exercises	See Examples
6–11, 14, 15	1
12, 13, 16–21	2
22–36	3

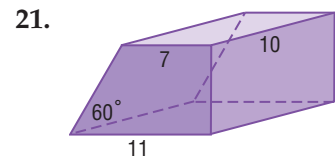
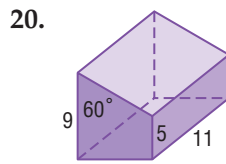
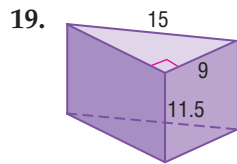
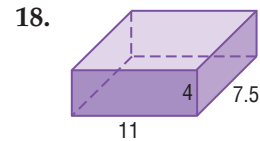
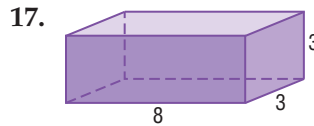
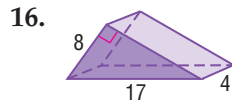
Extra Practice
See page 778.

Find the lateral area of each prism or solid. Round to the nearest tenth if necessary.



- The surface area of a cube is 864 square inches. Find the length of the lateral edge of the cube.
- The surface area of a triangular prism is 540 square centimeters. The bases are right triangles with legs measuring 12 centimeters and 5 centimeters. Find the height.
- The lateral area of a rectangular prism is 156 square inches. What are the possible dimensions of the prism if the height is 13 inches?
- The lateral area of a rectangular prism is 96 square meters. What are the possible dimensions of the prism if the height is 4 meters?

Find the surface area of each prism. Round to the nearest tenth if necessary.



PAINTING For Exercises 22–24, use the following information.

A gallon of paint costs \$16 and covers 400 square feet. Two coats of paint are recommended for even coverage. The room to be painted is 10 feet high, 15 feet long, and 15 feet wide. Only $1\frac{1}{2}$ gallons of paint are left to paint the room.

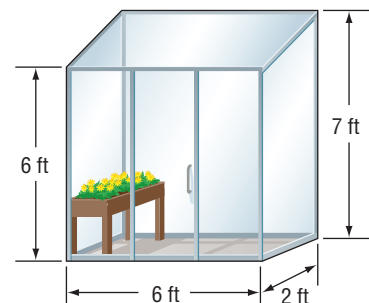
- Is this enough paint for the walls of the room? Explain.
- How many gallons of paint are needed to paint the walls?
- How much would it cost to paint the walls and ceiling?

TOURISM For Exercises 25–27, use the following information.

The World's Only Corn Palace is located in Mitchell, South Dakota. The sides of the building are covered with huge murals made from corn and other grains.

- Estimate the area of the Corn Palace to be covered if its base is 310 by 185 feet and it is 45 feet tall, not including the turrets.
- Suppose a bushel of grain can cover 15 square feet. How many bushels of grain does it take to cover the Corn Palace?
- Will the actual amount of grain needed be higher or lower than the estimate? Explain.

- GARDENING** This greenhouse is designed for a home gardener. It is attached to one wall of the house, and the frame is covered with tempered safety glass. Find the surface area of the glass covering the greenhouse.



More About . . .



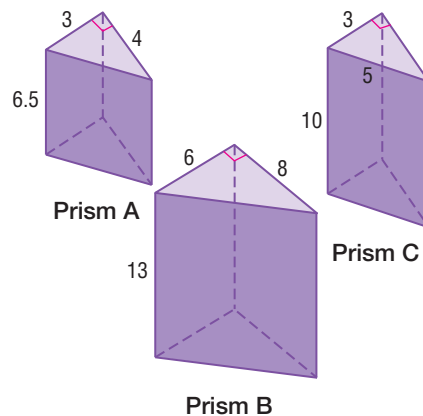
Tourism

Tourists visit the Corn Palace each year to see the murals. New murals are created each year by local artists. It costs approximately \$100,000 each year.

Source: www.cornpalace.org

For Exercises 29–33, use prisms A, B, and C.

29. Compare the bases of each prism.
30. Write three ratios to compare the perimeters of the bases of the prisms.
31. Write three ratios to compare the areas of the bases of the prisms.
32. Write three ratios to compare the surface areas of the prisms.
33. Which pairs of prisms have the same ratio of base areas as ratio of surface areas? Why do you think this is so?



STATISTICS For Exercises 34–36, use the graphic at the right.

Malik plans to build a three-dimensional model of the data from the graph.

- A rectangular prism will represent each category.
 - Each prism will be 30 centimeters wide and 20 centimeters deep.
 - The length of the prism for TV will be 84 centimeters.
34. Find the surface area of each prism that Malik builds.
 35. Will the surface area of the finished product be the sum of the surface areas of each prism? Explain.
 36. Find the total surface area of the finished model.
 37. **CRITICAL THINKING** Suppose the lateral area of a right rectangular prism is 144 square centimeters. If the length is three times the width and the height is twice the width, find the surface area.
 38. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do brick masons know how many bricks to order for a project?

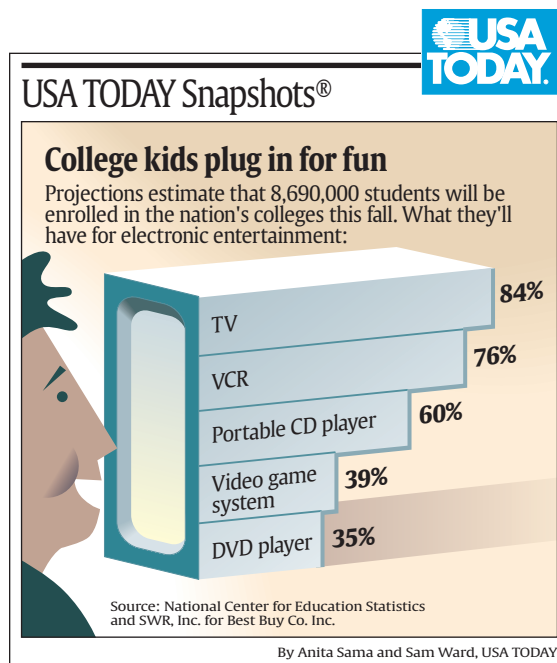
Include the following in your answer:

- how lateral area is used, and
- why overestimation is important in the process.

39. The surface area of a cube is 121.5 square meters. What is the length of each edge?

(A) 4.05 m (B) 4.5 m (C) 4.95 m (D) 5 m
40. **ALGEBRA** For all $a \neq 4$, $\frac{a^2 - 16}{4a - 16} = \underline{\hspace{2cm}}$.

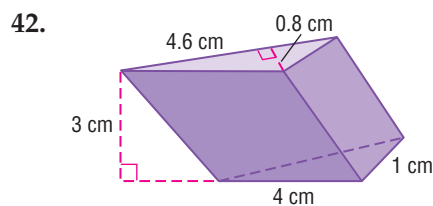
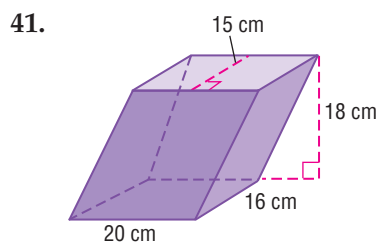
(A) $a + 16$ (B) $a + 1$ (C) $\frac{a - 4}{4}$ (D) $\frac{a + 4}{4}$



**Extending
the Lesson**

OBLIQUE PRISMS The difference between a right prism and an oblique prism is that the altitude of an oblique prism is not the length of a lateral edge. The lateral area and the surface area are computed in the same way as for right prisms.

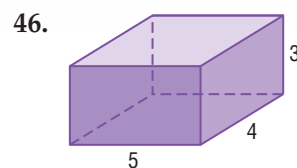
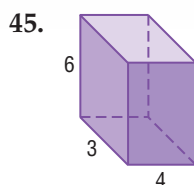
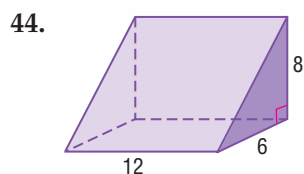
Find the lateral area and surface area of each oblique prism.



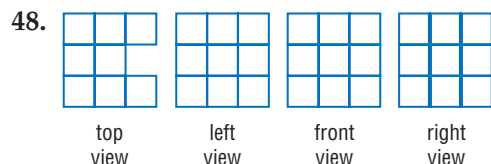
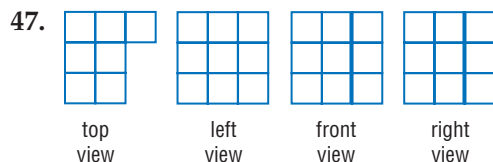
43. **RESEARCH** Use a dictionary to find the meaning of the term *oblique*. How is the everyday meaning related to the mathematical meaning?

Maintain Your Skills

Mixed Review For each solid, draw a net and find the surface area. (Lesson 12-2)

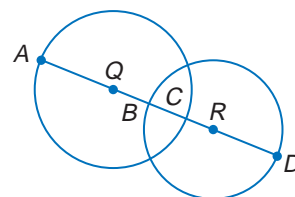


Draw the back view and corner view of the figure given the orthogonal drawing. (Lesson 12-1)



Circle Q has a radius of 24 units, $\odot R$ has a radius of 16 units, and $BC = 5$. Find each measure. (Lesson 10-1)

49. AB 50. AD 51. QR

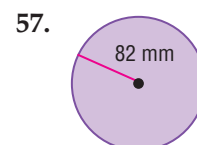
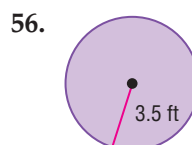
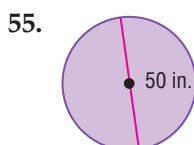
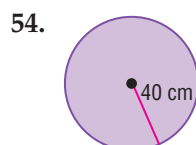


52. **NAVIGATION** An airplane is three miles above sea level when it begins to climb at a 3.5° angle. If this angle is constant, how far above sea level is the airplane after flying 50 miles? (Lesson 7-4)

53. **ART** Kiernan drew a sketch of a house. If the height of the house in her drawing was 5.5 inches and the actual height of the house was 33 feet, find the scale factor of the drawing. (Lesson 6-1)

**Getting Ready for
the Next Lesson**

PREREQUISITE SKILL Find the area of each circle. Round to the nearest hundredth. (To review *finding the area of a circle*, see Lesson 11-3.)



12-4

Surface Areas of Cylinders

What You'll Learn

- Find lateral areas of cylinders.
- Find surface areas of cylinders.

Vocabulary

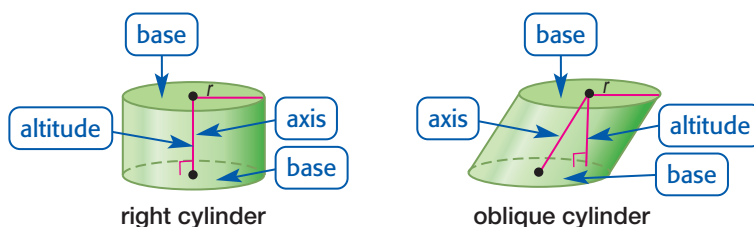
- axis
- right cylinder
- oblique cylinder

How are cylinders used in extreme sports?

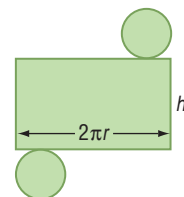
Extreme sports, such as in-line skating, biking, skateboarding, and snowboarding use a cylindrical-shaped ramp called a half-pipe. The half-pipe looks like half of a cylinder. Usually there is a flat section in the middle with sides almost 8 feet high. Near the top, the sides are almost vertical.



LATERAL AREAS OF CYLINDERS The **axis** of the cylinder is the segment with endpoints that are centers of the circular bases. If the axis is also the altitude, then the cylinder is called a **right cylinder**. Otherwise, the cylinder is an **oblique cylinder**.



The net of a cylinder is composed of two congruent circles and a rectangle. The area of this rectangle is the lateral area. The length of the rectangle is the same as the circumference of the base, $2\pi r$. So, the lateral area of a right cylinder is $2\pi rh$.



Study Tip

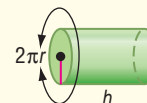
Formulas

An alternate formula for the lateral area of a cylinder is $L = \pi dh$, with πd as the circumference of a circle.

Key Concept

Lateral Area of a Cylinder

If a right cylinder has a lateral area of L square units, a height of h units, and the bases have radii of r units, then $L = 2\pi rh$.



Example 1 Lateral Area of a Cylinder

MANUFACTURING An office has recycling barrels for cans and paper. The barrels are cylindrical with cardboard sides and plastic lids and bases. Each barrel is 3 feet tall, and the diameter is 30 inches. How many square feet of cardboard are used to make each barrel?

The cardboard section of the barrel represents the lateral area of the cylinder. If the diameter of the lid is 30 inches, then the radius is 15 inches. The height is 3 feet or 36 inches. Use the formula to find the lateral area. *(continued on the next page)*

$$\begin{aligned}
 L &= 2\pi rh && \text{Lateral area of a cylinder} \\
 &= 2\pi(15)(36) && r = 15, h = 36 \\
 &\approx 3392.9 && \text{Use a calculator.}
 \end{aligned}$$

Each barrel uses approximately 3393 square inches of cardboard. Because 144 square inches equal one square foot, there are $3393 \div 144$ or about 23.6 square feet of cardboard per barrel.

SURFACE AREAS OF CYLINDERS To find the surface area of a cylinder, first find the lateral area and then add the areas of the bases. This leads to the formula for the surface area of a right cylinder.

Study Tip

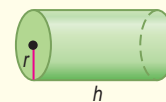
Making Connections

The formula for the surface area of a right cylinder is like that of a prism, $T = L + 2B$.

Key Concept

Surface Area of a Cylinder

If a right cylinder has a surface area of T square units, a height of h units, and the bases have radii of r units, then $T = 2\pi rh + 2\pi r^2$.

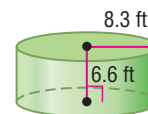


Example 2 Surface Area of a Cylinder

Find the surface area of the cylinder.

The radius of the base and the height of the cylinder are given. Substitute these values in the formula to find the surface area.

$$\begin{aligned}
 T &= 2\pi rh + 2\pi r^2 && \text{Surface area of a cylinder} \\
 &= 2\pi(8.3)(6.6) + 2\pi(8.3)^2 && r = 8.3, h = 6.6 \\
 &\approx 777.0 && \text{Use a calculator.}
 \end{aligned}$$



The surface area is approximately 777.0 square feet.

Example 3 Find Missing Dimensions

Find the radius of the base of a right cylinder if the surface area is 128π square centimeters and the height is 12 centimeters.

Use the formula for surface area to write and solve an equation for the radius.

$$\begin{aligned}
 T &= 2\pi rh + 2\pi r^2 && \text{Surface area of a cylinder} \\
 128\pi &= 2\pi(12)r + 2\pi r^2 && \text{Substitution} \\
 128\pi &= 24\pi r + 2\pi r^2 && \text{Simplify.} \\
 64 &= 12r + r^2 && \text{Divide each side by } 2\pi. \\
 0 &= r^2 + 12r - 64 && \text{Subtract 64 from each side.} \\
 0 &= (r - 4)(r + 16) && \text{Factor.} \\
 r &= 4 \text{ or } -16
 \end{aligned}$$

Since the radius of a circle cannot have a negative value, -16 is eliminated. So, the radius of the base is 4 centimeters.

Check for Understanding

Concept Check

1. Explain how to find the surface area of a cylinder.
2. **OPEN ENDED** Draw a net for a cylinder.
3. **FIND THE ERROR** Jamie and Dwayne are finding the surface area of a cylinder with one base.

Jamie

$$T = 2\pi(4)(9) + \pi(4^2)$$

$$T = 72\pi + 16\pi$$

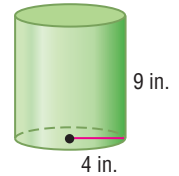
$$T = 88\pi \text{ in}^2$$

Dwayne

$$T = 2\pi(4)(9) + 2\pi(4^2)$$

$$T = 72\pi + 32\pi$$

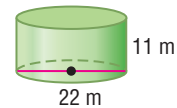
$$T = 104\pi \text{ in}^2$$



Who is correct? Explain.

Guided Practice

4. Find the surface area of a cylinder with a radius of 4 feet and height of 6 feet. Round to the nearest tenth.
5. Find the surface area of the cylinder. Round to the nearest tenth.



Find the radius of the base of each cylinder.

6. The surface area is 96π square centimeters, and the height is 8 centimeters.
7. The surface area is 140π square feet, and the height is 9 feet.

Concept Check

8. **CONTESTS** Mrs. Fairway's class is collecting labels from soup cans to raise money for the school. The students collected labels from 3258 cans. If the cans are 4 inches high with a diameter of 2.5 inches, find the area of the labels that were collected.

Practice and Apply

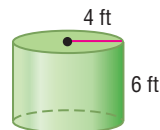
Concept Check

Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

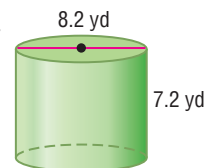
9. $r = 13 \text{ m}, h = 15.8 \text{ m}$
10. $d = 13.6 \text{ ft}, h = 1.9 \text{ ft}$
11. $d = 14.2 \text{ in.}, h = 4.5 \text{ in.}$
12. $r = 14 \text{ mm}, h = 14 \text{ mm}$

Find the surface area of each cylinder. Round to the nearest tenth.

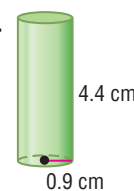
13.



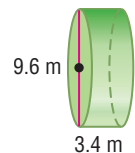
14.



15.



16.



Find the radius of the base of each cylinder.

17. The surface area is 48π square centimeters, and the height is 5 centimeters.
18. The surface area is 340π square inches, and the height is 7 inches.
19. The surface area is 320π square meters, and the height is 12 meters.
20. The surface area is 425.1 square feet, and the height is 6.8 feet.

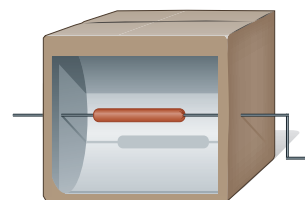
Homework Help

For Exercises	See Examples
9–16	2
17–20	3
21, 22, 14–25	1

Extra Practice
See page 779.

21. **KITCHENS** Raul purchased a set of canisters with diameters of 5 inches and heights of 9 inches, 6 inches, and 3 inches. Make a conjecture about the relationship between the heights of the canisters and their lateral areas. Check your conjecture.

22. **CAMPING** Campers can use a solar cooker to cook food. You can make a solar cooker from supplies you have on hand. The reflector in the cooker shown at the right is half of a cardboard cylinder covered with aluminum foil. The reflector is 18 inches long and has a diameter of $5\frac{1}{2}$ inches. How much aluminum foil was needed to cover the inside of the reflector?



23. Suppose the height of a right cylinder is tripled. Is the surface area or lateral area tripled? Explain.

- **AGRICULTURE** For Exercises 24 and 25, use the following information.

The acid from the contents of a silo can weaken its concrete walls and seriously damage the silo's structure. So the inside of the silo must occasionally be resurfaced. The cost of the resurfacing is a function of the lateral area of the inside of the silo.

24. Find the lateral area of a silo 13 meters tall with an interior diameter of 5 meters.

25. A second grain silo is 26 meters tall. If both silos have the same lateral area, find the radius of the second silo.

26. **CRITICAL THINKING** Some pencils are cylindrical, and others are hexagonal prisms. If the diameter of the cylinder is the same length as the longest diagonal of the hexagon, which has the greater surface area? Explain. Assume that each pencil is 11 inches long and unsharpened.

27. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are cylinders used in extreme sports?

Include the following in your answer:

- how to find the lateral area of a semicylinder, and
- how to determine if the half-pipe ramp is a semicylinder.

28. A cylinder has a height of 13.4 centimeters and a diameter of 8.2 centimeters. What is the surface area of the cylinder?

- (A) 51.5 cm^2 (B) 450.8 cm^2 (C) 741.9 cm^2 (D) 1112.9 cm^2

29. **ALGEBRA** For the band concert, student tickets cost \$2 and adult tickets cost \$5. A total of 200 tickets were sold. If the total sales were more than \$500, what was the minimum number of adult tickets sold?

- (A) 30 (B) 33 (C) 34 (D) 40

Extending the Lesson

LOCUS A cylinder can be defined in terms of locus. The locus of points in space a given distance from a line is a cylinder.

Draw a figure and describe the locus of all points in space that satisfy each set of conditions.

30. 5 units from a given line

31. equidistant from two opposite vertices of a face of a cube



Career Choices

Agricultural Engineer

Agricultural engineers use engineering techniques to develop equipment and methods that assist farmers in efficient agricultural production.

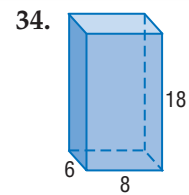
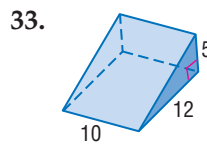
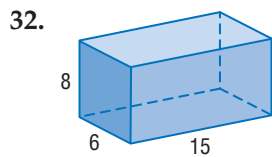
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Standardized Test Practice

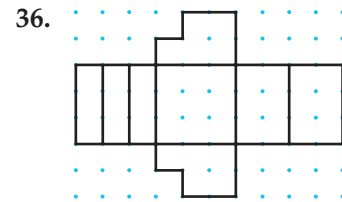
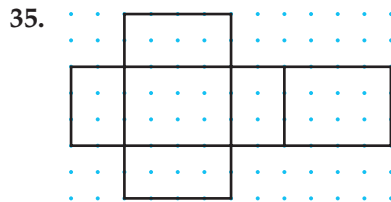
(A) (B) (C) (D)

Maintain Your Skills

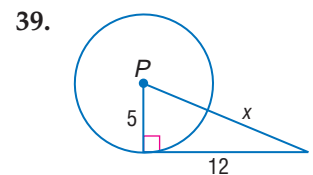
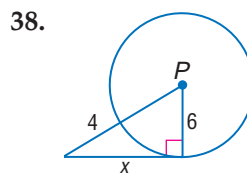
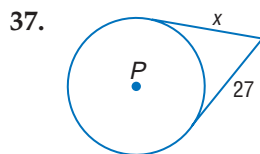
Mixed Review Find the lateral area of each prism. (Lesson 12-3)



Given the net of a solid, use isometric dot paper to draw the solid. (Lesson 12-2)



Find x . Assume that segments that appear to be tangent are tangent. (Lesson 10-5)



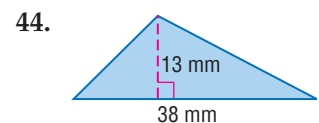
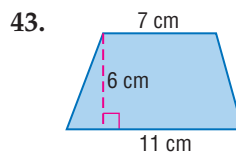
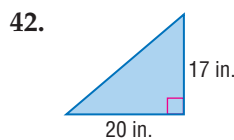
Solve each $\triangle ABC$ described below. Round to the nearest tenth if necessary. (Lesson 7-7)

40. $m\angle A = 54$, $b = 6.3$, $c = 7.1$

41. $m\angle B = 47$, $m\angle C = 69$, $a = 15$

Getting Ready for the Next Lesson

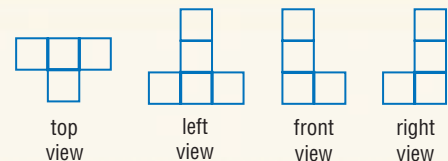
PREREQUISITE SKILL Find the area of each figure. (To review finding areas of triangles and trapezoids, see Lesson 11-2.)



Practice Quiz 1

Lessons 12-1 through 12-4

1. Draw a corner view of the figure given the orthogonal drawing. (Lesson 12-1)

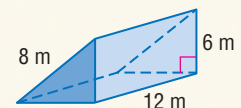


2. Sketch a rectangular prism 2 units wide, 3 units long, and 2 units high using isometric dot paper. (Lesson 12-2)

3. Find the lateral area of the prism. Round to the nearest tenth. (Lesson 12-3)

4. Find the surface area of the prism. Round to the nearest tenth. (Lesson 12-4)

5. Find the radius of the base of a right cylinder if the surface area is 560 square feet and the height is 11 feet. Round to the nearest tenth. (Lesson 12-4)



12-5 Surface Areas of Pyramids

What You'll Learn

- Find lateral areas of regular pyramids.
- Find surface areas of regular pyramids.

Vocabulary

- regular pyramid
- slant height

How are pyramids used in architecture?

In 1989, a new entrance was completed in the courtyard of the Louvre museum in Paris, France. Visitors can enter the museum through a glass pyramid that stands 71 feet tall. The pyramid is glass with a structural system of steel rods and cables.



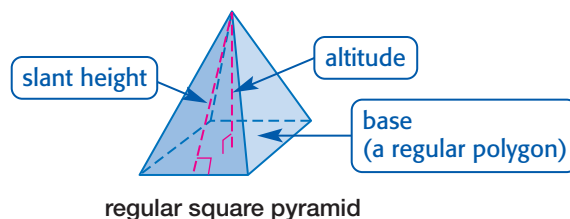
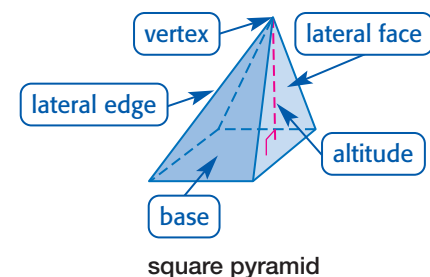
LATERAL AREAS OF REGULAR PYRAMIDS Pyramids have the following characteristics.

- All of the faces, except the base, intersect at one point called the *vertex*.
- The base is always a polygon.
- The faces that intersect at the vertex are called *lateral faces* and form triangles. The edges of the lateral faces that have the vertex as an endpoint are called *lateral edges*.
- The *altitude* is the segment from the vertex perpendicular to the base.

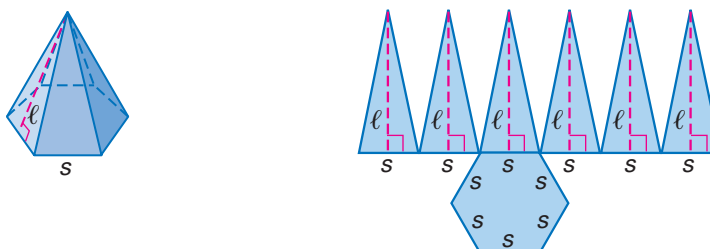
Study Tip

Right Pyramid

In a *right pyramid*, the altitude is the segment with endpoints that are the center of the base and the vertex. But the base is not always a regular polygon.



The figure below is a regular hexagonal pyramid. Its lateral area L can be found by adding the areas of all its congruent triangular faces as shown in its net.



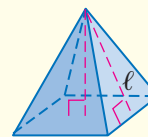
Area of the net

$$\begin{aligned}
 L &= \frac{1}{2}s\ell + \frac{1}{2}s\ell + \frac{1}{2}s\ell + \frac{1}{2}s\ell + \frac{1}{2}s\ell + \frac{1}{2}s\ell && \text{Sum of the areas of the lateral faces} \\
 &= \frac{1}{2}\ell(s + s + s + s + s + s) && \text{Distributive Property} \\
 &= \frac{1}{2}P\ell && P = s + s + s + s + s + s
 \end{aligned}$$

Key Concept

Lateral Area of a Regular Pyramid

If a regular pyramid has a lateral area of L square units, a slant height of ℓ units, and its base has a perimeter of P units, then $L = \frac{1}{2}P\ell$.



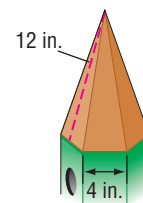
Example 1 Use Lateral Area to Solve a Problem

BIRDHOUSES The roof of a birdhouse is a regular hexagonal pyramid. The base of the pyramid has sides of 4 inches, and the slant height of the roof is 12 inches. If the roof is made of copper, find the amount of copper used for the roof.

We need to find the lateral area of the hexagonal pyramid. The sides of the base measure 4 inches, so the perimeter is $6(4)$ or 24 inches.

$$\begin{aligned}
 L &= \frac{1}{2}P\ell && \text{Lateral area of a regular pyramid} \\
 &= \frac{1}{2}(24)(12) && P = 24, \ell = 12 \\
 &= 144 && \text{Multiply.}
 \end{aligned}$$

So, 144 square inches of copper are used to cover the roof of the birdhouse.



Study Tip

Making Connections

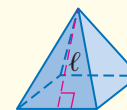
The total surface area for a pyramid is $L + B$, because there is only one base to consider.

SURFACE AREAS OF REGULAR PYRAMIDS The surface area of a regular pyramid is the sum of the lateral area and the area of the base.

Key Concept

Surface Area of a Regular Pyramid

If a regular pyramid has a surface area of T square units, a slant height of ℓ units, and its base has a perimeter of P units and an area of B square units, then $T = \frac{1}{2}P\ell + B$.

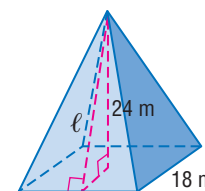


Example 2 Surface Area of a Square Pyramid

Find the surface area of the square pyramid.

To find the surface area, first find the slant height of the pyramid. The slant height is the hypotenuse of a right triangle with legs that are the altitude and a segment with a length that is one-half the side measure of the base.

$$\begin{aligned}
 c^2 &= a^2 + b^2 && \text{Pythagorean Theorem} \\
 \ell^2 &= 9^2 + 24^2 && a = 9, b = 24, c = \ell \\
 \ell &= \sqrt{657} && \text{Simplify.}
 \end{aligned}$$



(continued on the next page)



Study Tip

Look Back

To review **finding the areas of regular polygons**, see Lesson 11-3. To review **trigonometric ratios**, see Lesson 7-4.

Now find the surface area of a regular pyramid. The perimeter of the base is $4(18)$ or 72 meters, and the area of the base is 18^2 or 324 square meters.

$$T = \frac{1}{2}P\ell + B \quad \text{Surface area of a regular pyramid}$$

$$T = \frac{1}{2}(72)\sqrt{657} + 324 \quad P = 72, \ell = \sqrt{657}, B = 324$$

$$T \approx 1246.8 \quad \text{Use a calculator.}$$

The surface area is 1246.8 square meters to the nearest tenth.

Example 3 Surface Area of Pentagonal Pyramid

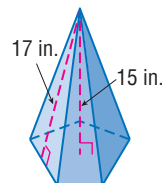
Find the surface area of the regular pyramid.

The altitude, slant height, and apothem form a right triangle. Use the Pythagorean Theorem to find the apothem. Let a represent the length of the apothem.

$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

$$(17)^2 = a^2 + 15^2 \quad b = 15, c = 17$$

$$8 = a \quad \text{Simplify.}$$



Now find the length of the sides of the base. The central angle of the pentagon measures $\frac{360^\circ}{5}$ or 72° . Let x represent the measure of the angle formed by a radius and the apothem. Then, $x = \frac{72}{2}$ or 36 .

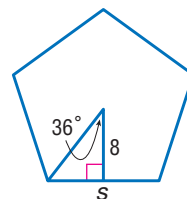
Use trigonometry to find the length of the sides.

$$\tan 36^\circ = \frac{\frac{1}{2}s}{8} \quad \tan x^\circ = \frac{\text{opposite}}{\text{adjacent}}$$

$$8(\tan 36^\circ) = \frac{1}{2}s \quad \text{Multiply each side by 8.}$$

$$16(\tan 36^\circ) = s \quad \text{Multiply each side by 2.}$$

$$11.6 \approx s \quad \text{Use a calculator.}$$



Next, find the perimeter and area of the base.

$$P = 5s \\ \approx 5(11.6) \text{ or } 58$$

$$B = \frac{1}{2}Pa \\ \approx \frac{1}{2}(58)(8) \text{ or } 232$$

Finally, find the surface area.

$$T = \frac{1}{2}P\ell + B \quad \text{Surface area of a regular pyramid}$$

$$\approx \frac{1}{2}(58)(17) + 232 \quad P \approx 58, \ell = 17, B \approx 232$$

$$\approx 726.5 \quad \text{Simplify.}$$

The surface area is approximately 726.5 square inches.

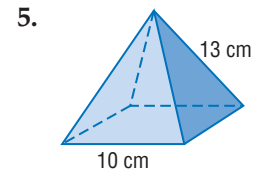
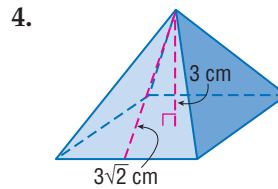
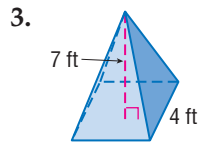
WebQuest

Making a sketch of a pyramid can help you find its slant height, lateral area, and base area. Visit www.geometryonline.com/webquest to continue work on your WebQuest project.

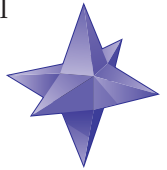
Check for Understanding

- Concept Check**
- OPEN ENDED** Draw a regular pyramid and a pyramid that is not regular.
 - Explain** whether a regular pyramid can also be a regular polyhedron.

Guided Practice Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.



- Application**
- DECORATIONS** Minowa purchased 3 decorative three-dimensional stars. Each star is composed of 6 congruent square pyramids with faces of paper and a base of cardboard. If the base is 2 inches on each side and the slant height is 4 inches, find the amount of paper used for one star.



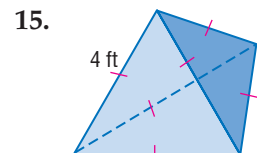
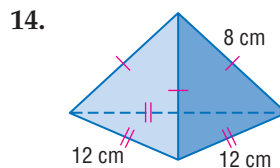
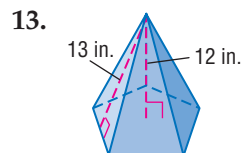
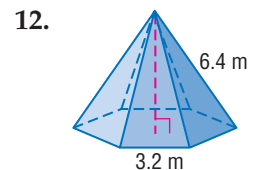
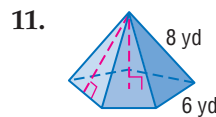
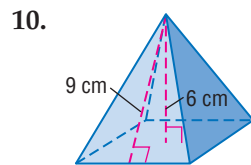
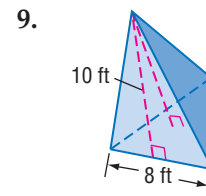
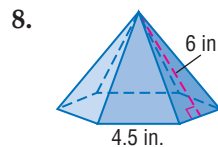
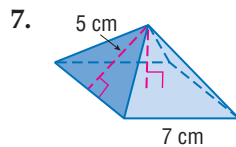
Practice and Apply

Homework Help

For Exercises	See Examples
7, 9, 10, 14, 15	2
8, 11–13	3
16, 18–24	1

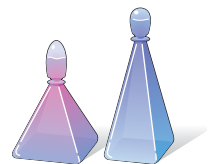
Extra Practice
See page 779.

Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.



- CONSTRUCTION** The roof on a building is a square pyramid. If the altitude of the pyramid measures 5 feet and the slant height measures 20 feet, find the area of the roof.

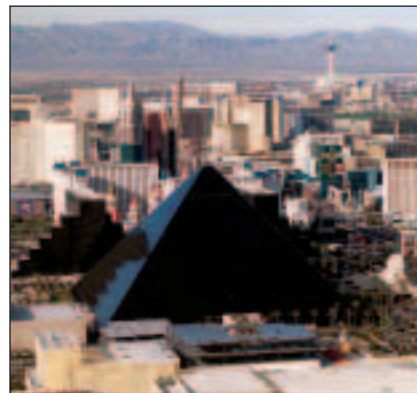
- PERFUME BOTTLES** Some perfumes are packaged in square pyramidal containers. The base of one bottle is 3 inches square, and the slant height is 4 inches. A second bottle has the same surface area, but the slant height is 6 inches long. Find the dimensions of the base of the second bottle.



18. **STADIUMS** The Pyramid Arena in Memphis, Tennessee, is the third largest pyramid in the world. The base is 360,000 square feet, and the pyramid is 321 feet tall. Find the lateral area of the pyramid. (Assume that the base is a square).



19. **HOTELS** The Luxor Hotel in Las Vegas is a black glass pyramid. The base is a square with edges 646 feet long. The hotel is 350 feet tall. Find the area of the glass.



More About . . .



History

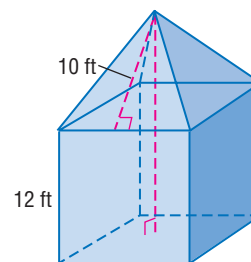
Egyptologists believe that the Great Pyramids of Egypt were originally covered with white limestone that has worn away or been removed.

Source: www.pbs.org

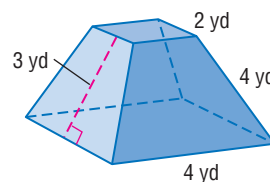
20. **HISTORY** Each side of the square base of Khafre's Pyramid is 214.5 meters. The sides rise at an angle of about 53° . Find the lateral area of the pyramid.

For Exercises 21–23, use the following information. This solid is a composite of a cube and square pyramid. The base of the solid is the base of the cube. Find the indicated measurements for the solid.

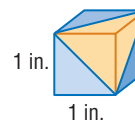
21. Find the height.
22. Find the lateral area.
23. Find the surface area.



24. A *frustum* is the part of a solid that remains after the top portion has been cut by a plane parallel to the base. Find the lateral area of the frustum of a regular pyramid.



25. **CRITICAL THINKING** This square prism measures 1 inch on each side. The corner of the cube is cut off, or *truncated*. Does this change the surface area of the cube? Include the surface area of the original cube and that of the truncated cube in your answer.



26. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are pyramids used in architecture?

Include the following in your answer:

- information needed to find the lateral area and surface area, and
- other examples of pyramidal shapes used in architecture.

27. The base of a square pyramid has a perimeter of 20 centimeters, and the slant height is 10 centimeters. What is the surface area of the pyramid?

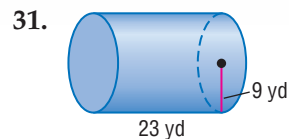
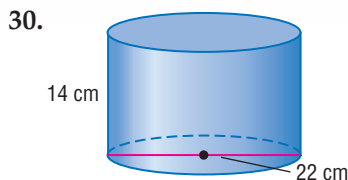
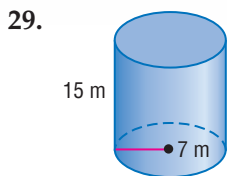
- (A) 96.8 cm^2 (B) 116 cm^2 (C) 121.8 cm^2 (D) 125 cm^2

28. **ALGEBRA** If $x \otimes y = \frac{1}{x-y}$, what is the value of $\frac{1}{2} \otimes \frac{3}{4}$?

- (A) -4 (B) $-\frac{1}{4}$ (C) $\frac{4}{5}$ (D) $\frac{5}{4}$

Maintain Your Skills

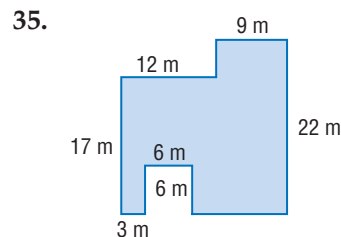
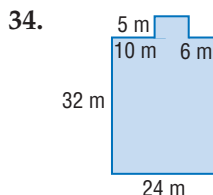
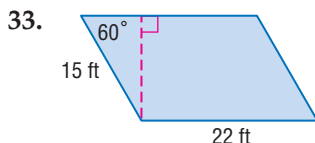
Mixed Review Find the surface area of each cylinder. Round to the nearest tenth. (Lesson 12-4)



32. **FOOD** Most cereals are packaged in cardboard boxes. If a box of cereal is 14 inches high, 6 inches wide, and 2.5 inches deep, find the surface area of the box. (Lesson 12-3)

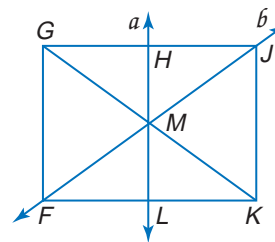


Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary. (Lesson 11-1)



For Exercises 36–39, refer to figure below. Name the reflected image of each figure. (Lesson 9-1)

36. \overline{FM} in line b
 37. \overline{JK} in line a
 38. L in point M
 39. GM in line a



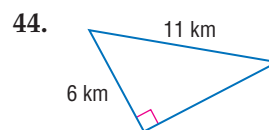
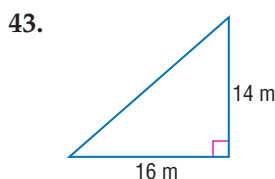
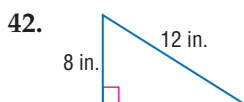
Determine whether each statement is *true* or *false*. Explain. (Lesson 8-3)

40. If two pairs of consecutive sides of a quadrilateral are congruent, then the quadrilateral must be a parallelogram.
 41. If all four sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Getting Ready for the Next Lesson

PREREQUISITE SKILL Use the Pythagorean Theorem to solve for the missing length in each triangle. Round to the nearest tenth.

(To review the **Pythagorean Theorem**, see Lesson 7-2.)



12-6 Surface Areas of Cones

What You'll Learn

- Find lateral areas of cones.
- Find surface areas of cones.

Vocabulary

- circular cone
- right cone
- oblique cone

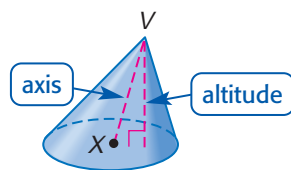
How is the lateral area of a cone used to cover tepees?

Native American tribes on the Great Plains typically lived in tepees, or tipis. Tent poles were arranged in a conical shape, and animal skins were stretched over the frame for shelter. The top of a tepee was left open for smoke to escape.

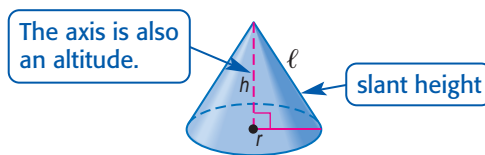


LATERAL AREAS OF CONES The shape of a tepee suggests a **circular cone**. Cones have the following characteristics.

- The base is a circle and the vertex is the point V .
- The *axis* is the segment with endpoints that are the vertex and the center of the base.
- The segment that has the vertex as one endpoint and is perpendicular to the base is called the *altitude* of the cone.



oblique cone



right cone

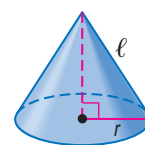
Study Tip

Reading Math

From this point in the text, you can assume that cones are right circular cones. If the cone is oblique, it will be clearly stated.

A cone with an axis that is also an altitude is called a **right cone**. Otherwise, it is called an **oblique cone**. The measure of any segment joining the vertex of a right cone to the edge of the circular base is called the *slant height*, ℓ . The measure of the altitude is the height h of the cone.

We can use the net for a cone to derive the formula for the lateral area of a cone. The lateral region of the cone is a sector of a circle with radius ℓ , the slant height of the cone. The arc length of the sector is the same as the circumference of the base, or $2\pi r$. The circumference of the circle containing the sector is $2\pi\ell$. The area of the sector is proportional to the area of the circle.



$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{measure of arc}}{\text{circumference of circle}}$$

Write a proportion.

$$\frac{\text{area of sector}}{\pi\ell^2} = \frac{2\pi r}{2\pi\ell}$$

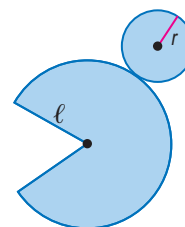
Substitution

$$\text{area of sector} = \frac{(\pi\ell^2)(2\pi r)}{2\pi\ell}$$

Multiply each side by $\pi\ell^2$.

$$\text{area of sector} = \pi r\ell$$

Simplify.



This derivation leads to the formula for the lateral area of a right circular cone.

Key Concept

Lateral Area of a Cone

If a right circular cone has a lateral area of L square units, a slant height of ℓ units, and the radius of the base is r units, then $L = \pi r \ell$.



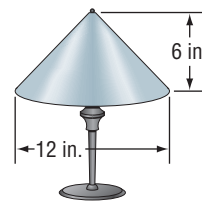
Study Tip

Storing Values in Calculator Memory

A graphing calculator will let you store values for future use. For example, you could store the calculated value of ℓ by $\sqrt{\square}$ 72 $\text{STO} \blacktriangleright$ ALPHA [L]. To find the lateral area, use 2nd [π] \times 6 \times ALPHA [L] ENTER .

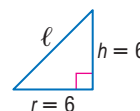
Example 1 Lateral Area of a Cone

LAMPS Diego has a conical lamp shade with an altitude of 6 inches and a diameter of 12 inches. Find the lateral area of the lampshade.



Explore We are given the altitude and the diameter of the base. We need to find the slant height of the cone.

Plan The radius of the base, height, and slant height form a right triangle. Use the Pythagorean Theorem to solve for the slant height. Then use the formula for the lateral area of a right circular cone.



Solve Write an equation and solve for ℓ .

$$\ell^2 = 6^2 + 6^2 \quad \text{Pythagorean Theorem}$$

$$\ell^2 = 72 \quad \text{Simplify.}$$

$$\ell = \sqrt{72} \text{ or } 6\sqrt{2} \quad \text{Take the square root of each side.}$$

Next, use the formula for the lateral area of a right circular cone.

$$L = \pi r \ell \quad \text{Lateral area of a cone}$$

$$\approx \pi(6)(6\sqrt{2}) \quad r = 6, \ell = 6\sqrt{2}$$

$$\approx 159.9 \quad \text{Use a calculator.}$$

The lateral area is approximately 159.9 square inches.

Examine Use estimation to check the reasonableness of this result. The lateral area is approximately $3 \cdot 6 \cdot 9$ or 162 square inches. Compared to the estimate, the answer is reasonable.

SURFACE AREAS OF CONES To find the surface area of a cone, add the area of the base to the lateral area.

Study Tip

Making Connections

The surface area of a cone is like the surface area of a pyramid, $T = L + B$.

Key Concept

Surface Area of a Cone

If a right circular cone has a surface area of T square units, a slant height of ℓ units, and the radius of the base is r units, then $T = \pi r \ell + \pi r^2$.



Example 2 Surface Area of a Cone

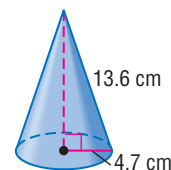
Find the surface area of the cone.

$$T = \pi r \ell + \pi r^2 \quad \text{Surface area of a cone}$$

$$= \pi(4.7)(13.6) + \pi(4.7)^2 \quad r = 4.7, \ell = 13.6$$

$$\approx 270.2 \quad \text{Use a calculator.}$$

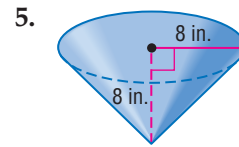
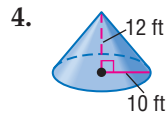
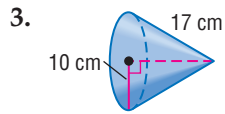
The surface area is approximately 270.2 square centimeters.



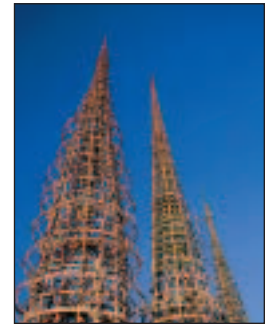
Check for Understanding

- Concept Check**
- OPEN ENDED** Draw an oblique cone. Mark the vertex and the center of the base.
 - Explain** why the formula for the lateral area of a right circular cone does not apply to oblique cones.

Guided Practice Find the surface area of each cone. Round to the nearest tenth.



- Application**
- TOWERS** In 1921, Italian immigrant Simon Rodia bought a home in Los Angeles, California, and began building conical towers in his backyard. The structures are made of steel mesh and cement mortar. Suppose the height of one tower is 55 feet and the diameter of the base is 8.5 feet, find the lateral area of the tower.



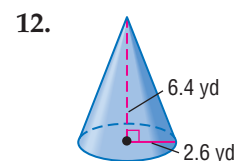
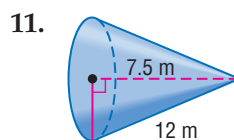
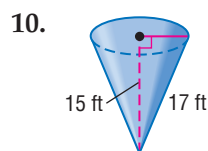
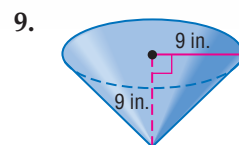
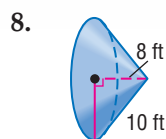
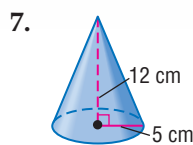
Practice and Apply

Homework Help

For Exercises	See Examples
7–21, 31	2
22–30	1

Extra Practice
See page 779.

Find the surface area of each cone. Round to the nearest tenth.



For Exercises 13–16, round to the nearest tenth.

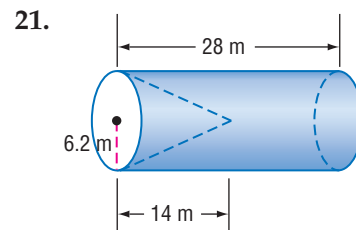
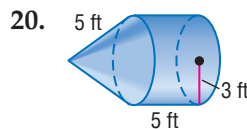
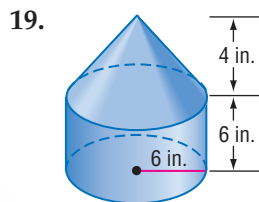
- Find the surface area of the cone if the height is 16 inches and the slant height is 18 inches.
- Find the surface area of the cone if the height is 8.7 meters and the slant height is 19.1 meters.
- The surface area of a cone is 1020 square meters and the radius is 14.5 meters. Find the slant height.
- The surface area of a cone is 293.2 square feet and the radius is 6.1 feet. Find the slant height.

Find the radius of a cone given the surface area and slant height. Round to the nearest tenth.

17. $T = 359 \text{ ft}^2$, $\ell = 15 \text{ ft}$

18. $T = 523 \text{ m}^2$, $\ell = 12.1 \text{ m}$

Find the surface area of each solid. Round to the nearest tenth.



More About...

Tepees

The Saamis Tepee in Medicine Hat, Alberta, Canada, is the world's largest tepee. The frame is made of steel, instead of wood. It weighs 200 tons.

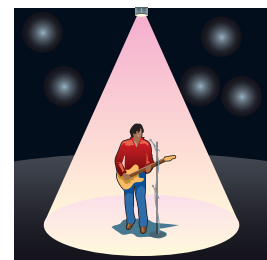
Source:
www.medicinehatchamber.com

22. **TEPEES** Find the area of canvas used to cover a tepee if the diameter of the base is 42 feet and the slant height is 47.9 feet.

23. **PARTY HATS** Shelley plans to make eight conical party hats for her niece's birthday party. She wants each hat to be 18 inches tall and the bases of each to be 22 inches in circumference. How much material will she use to make the hats?

24. **WINTER STORMS** Many states use a cone structure to store salt used to melt snow on highways and roads. Find the lateral area of one of these cone structures if the building measures 24 feet tall and the diameter is 45 feet.

25. **SPOTLIGHTS** A yellow-pink spotlight was positioned directly above a performer. If the surface area of the cone of light was approximately 500 square feet and the slant height was 20 feet, find the diameter of light on stage.



The height of a cone is 7 inches, and the radius is 4 inches. Round final answers to the nearest ten-thousandth.

26. Find the lateral area of the cone using the store feature of a calculator.
27. Round the slant height to the nearest tenth and then calculate the lateral area of the cone.
28. Round the slant height to the nearest hundredth and then calculate the lateral area of the cone.
29. Compare the lateral areas for Exercises 26–28. Which is most accurate? Explain.

Determine whether each statement is *sometimes*, *always*, or *never* true. Explain.

30. If the diagonal of the base of a square pyramid is equal to the diameter of the base of a cone and the heights of both solids are equal, then the pyramid and cone have equal lateral areas.
31. If the radii of the bases of two cones are proportional, then the surface areas are proportional.
32. **CRITICAL THINKING** If you were to move the vertex of a right cone down the axis toward the center of the base, explain what would happen to the lateral area and surface area of the cone.
33. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

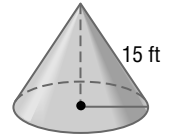
How is the lateral area of a cone used to cover tepees?

Include the following in your answer:

- information needed to find the lateral area of the canvas covering, and
- how the open top of a tepee affects the lateral area of the canvas covering it.



34. The lateral area of the cone is 91.5π square feet. What is the radius of the base?
 (A) 5.9 ft (B) 6.1 ft (C) 7.5 ft (D) 10 ft
35. **ALGEBRA** Three times the first of three consecutive odd integers is 3 more than twice the third. Find the third integer.
 (A) 9 (B) 11 (C) 13 (D) 15



Maintain Your Skills

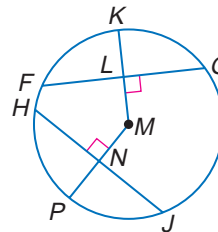
- Mixed Review** 36. **ARCHITECTURE** The Transamerica Tower in San Francisco is a regular pyramid with a square base that is 149 feet on each side and a height of 853 feet. Find its lateral area. (Lesson 12-5)

Find the radius of the base of the right cylinder. Round to the nearest tenth.
 (Lesson 12-4)

37. The surface area is 563 square feet, and the height is 9.5 feet.
 38. The surface area is 185 square meters, and the height is 11 meters.
 39. The surface area is 470 square yards, and the height is 6.5 yards.
 40. The surface area is 951 square centimeters, and the height is 14 centimeters.

In $\odot M$, $FL = 24$, $HJ = 48$, and $m\widehat{HP} = 45$.
 Find each measure. (Lesson 10-3)

41. FG 42. NJ
 43. HN 44. LG
 45. $m\widehat{PJ}$ 46. $m\widehat{HJ}$



Find the geometric mean between each pair of numbers. (Lesson 7-1)

47. 7 and 63 48. 8 and 18 49. 16 and 44

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find the circumference of each circle given the radius or the diameter. Round to the nearest tenth.

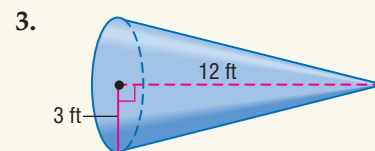
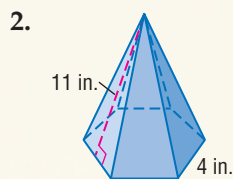
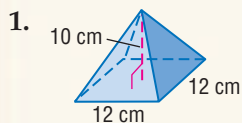
(To review finding the circumference of a circle, see Lesson 10-1.)

50. $r = 6$ 51. $d = 8$ 52. $d = 18$
 53. $r = 8.2$ 54. $d = 19.8$ 55. $r = 4.1$

Practice Quiz 2

Lessons 12-5 and 12-6

Find the surface area of each solid. Round to the nearest tenth. (Lessons 12-5 and 12-6)



4. Find the surface area of a cone if the radius is 6 meters and the height is 2 meters. Round to the nearest tenth. (Lesson 12-6)
5. Find the slant height of a cone if the lateral area is 123 square inches and the radius is 10 inches. Round to the nearest tenth. (Lesson 12-6)

What You'll Learn

- Recognize and define basic properties of spheres.
- Find surface areas of spheres.

Vocabulary

- great circle
- hemisphere

How do manufacturers of sports equipment use the surface areas of spheres?

The sports equipment industry has grown significantly in recent years because people are engaging in more physical activities. Balls are used in many sports, such as golf, basketball, baseball, and soccer. Some are hollow, while others have solid inner cores. Each ball is identifiable by its design, texture, color, and size.



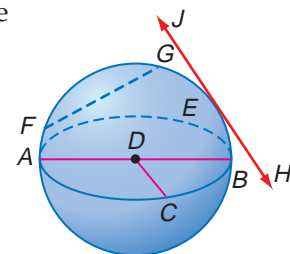
PROPERTIES OF SPHERES To visualize a sphere, consider infinitely many congruent circles in space, all with the same point for their center. Considered together, these circles form a sphere. In space, a sphere is the locus of all points that are a given distance from a given point called its *center*.

**Study Tip****Circles and Spheres**

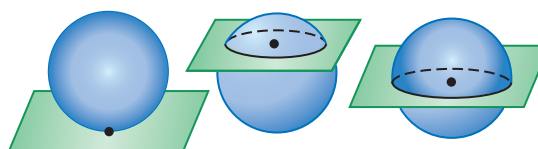
The shortest distance between any two points on a sphere is the length of the arc of a great circle passing through those two points.

There are several special segments and lines related to spheres.

- A segment with endpoints that are the center of the sphere and a point on the sphere is a *radius* of the sphere. In the figure, \overline{DC} , \overline{DA} , and \overline{DB} are radii.
- A *chord* of a sphere is a segment with endpoints that are points on the sphere. In the figure, \overline{CF} and \overline{AB} are chords.
- A chord that contains the center of the sphere is a *diameter* of the sphere. In the figure, \overline{AB} is a diameter.
- A *tangent* to a sphere is a line that intersects the sphere in exactly one point. In the figure, \overline{JH} is tangent to the sphere at E .



The intersection of a plane and a sphere can be a point or a circle. When a plane intersects a sphere so that it contains the center of the sphere, the intersection is called a **great circle**. A great circle has the same center as the sphere, and its radii are also radii of the sphere.



a point

a circle

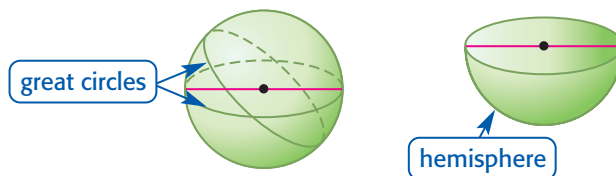
a great circle

Study Tip

Great Circles

A sphere has an infinite number of great circles.

Each great circle separates a sphere into two congruent halves, each called a **hemisphere**.



Example 1 Spheres and Circles

In the figure, O is the center of the sphere, and plane \mathcal{R} intersects the sphere in circle A . If $AO = 3$ centimeters and $OB = 10$ centimeters, find AB .

The radius of circle A is the segment \overline{AB} , B is a point on circle A and on sphere O . Use the Pythagorean Theorem for right triangle ABO to solve for AB .

$$OB^2 = AB^2 + AO^2 \quad \text{Pythagorean Theorem}$$

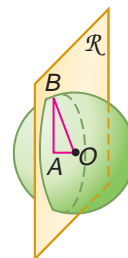
$$10^2 = AB^2 + 3^2 \quad OB = 10, AO = 3$$

$$100 = AB^2 + 9 \quad \text{Simplify.}$$

$$91 = AB^2 \quad \text{Subtract 9 from each side.}$$

$$9.5 \approx AB \quad \text{Use a calculator.}$$

AB is approximately 9.5 centimeters.



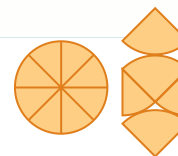
SURFACE AREAS OF SPHERES You will investigate the surface area of a sphere in the geometry activity.

Geometry Activity

Surface Area of a Sphere

Model

- Cut a polystyrene ball along a great circle. Trace the great circle onto a piece of paper. Then cut out the circle.
- Fold the circle into eight sectors. Then unfold and cut the pieces apart. Tape the pieces back together in the pattern shown at the right.
- Use tape or glue to put the two pieces of the ball together. Tape the paper pattern to the sphere.



Analyze

1. Approximately what fraction of the surface of the sphere is covered by the pattern?
2. What is the area of the pattern in terms of r , the radius of the sphere?

Make a Conjecture

3. Make a conjecture about the formula for the surface area of a sphere.



The activity leads us to the formula for the surface area of a sphere.

Key Concept

Surface Area of a Sphere

If a sphere has a surface area of T square units and a radius of r units, then $T = 4\pi r^2$.



Example 2 Surface Area

- a. Find the surface area of the sphere given the area of the great circle.

From the activity, we find that the surface area of a sphere is four times the area of the great circle.

$$T = 4\pi r^2 \quad \text{Surface area of a sphere}$$

$$\approx 4(201.1) \quad \pi r^2 \approx 201.1$$

$$\approx 804.4 \quad \text{Multiply.}$$

The surface area is approximately 804.4 square inches.



$$A \approx 201.1 \text{ in}^2$$

- b. Find the surface area of the hemisphere.

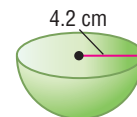
A hemisphere is half of a sphere. To find the surface area, find half of the surface area of the sphere and add the area of the great circle.

$$\text{surface area} = \frac{1}{2}(4\pi r^2) + \pi r^2 \quad \text{Surface area of a hemisphere}$$

$$= \frac{1}{2}[4\pi(4.2)^2] + \pi(4.2)^2 \quad \text{Substitution}$$

$$\approx 166.3 \quad \text{Use a calculator.}$$

The surface area is approximately 166.3 square centimeters.



More About . . .



Baseball

A great circle of a standard baseball has a circumference between 9 and $9\frac{1}{4}$ inches.

Source: www.mlb.com

Example 3 Surface Area

- **BASEBALL** Find the surface area of a baseball with a circumference of 9 inches to determine how much leather is needed to cover the ball.

First, find the radius of the sphere.

$$C = 2\pi r \quad \text{Circumference of a circle}$$

$$9 = 2\pi r \quad C = 9$$

$$\frac{9}{2\pi} = r \quad \text{Divide each side by } 2\pi.$$

$$1.4 \approx r \quad \text{Use a calculator.}$$

Next, find the surface area of a sphere.

$$T = 4\pi r^2 \quad \text{Surface area of a sphere}$$

$$\approx 4\pi(1.4)^2 \quad r \approx 1.4$$

$$\approx 25.8 \quad \text{Use a calculator.}$$

The surface area is approximately 25.8 square inches.



Check for Understanding

Concept Check

- OPEN ENDED** Draw a sphere and a great circle.
- FIND THE ERROR** Loesha and Tim are finding the surface area of a hemisphere with a radius of 6 centimeters.

Loesha

$$T = \frac{1}{2}(4\pi r^2)$$

$$T = 2\pi(6^2)$$

$$T = 72\pi$$

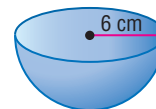
Tim

$$T = \frac{1}{2}(4\pi r^2) + \pi r^2$$

$$T = 2\pi(6^2) + \pi(6^2)$$

$$T = 72\pi + 36\pi$$

$$T = 108\pi$$

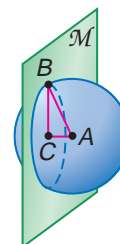


Who is correct? Explain.

Guided Practice

In the figure, A is the center of the sphere, and plane \mathcal{M} intersects the sphere in circle C .

- If $AC = 9$ and $BC = 12$, find AB .
- If the radius of the sphere is 15 units and the radius of the circle is 10 units, find AC .
- If Q is a point on $\odot C$ and $AB = 18$, find AQ .



Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

- a sphere with radius 6.8 inches
- a hemisphere with the circumference of a great circle 8π centimeters
- a sphere with the area of a great circle approximately 18.1 square meters

Application

- BASKETBALL** An NCAA (National Collegiate Athletic Association) basketball has a radius of $4\frac{3}{4}$ inches. Find the surface area.

Practice and Apply

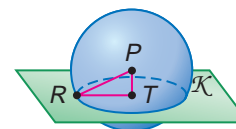
Homework Help

For Exercises	See Examples
10–15, 25–29	1
16, 30–38	3
17–24, 39, 40	2

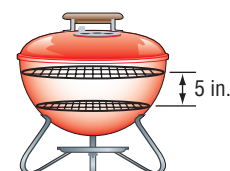
Extra Practice
See page 780.

In the figure, P is the center of the sphere, and plane \mathcal{K} intersects the sphere in circle T .

- If $PT = 4$ and $RT = 3$, find PR .
- If $PT = 3$ and $RT = 8$, find PR .
- If the radius of the sphere is 13 units and the radius of $\odot T$ is 12 units, find PT .
- If the radius of the sphere is 17 units and the radius of $\odot T$ is 15 units, find PT .
- If X is a point on $\odot T$ and $PR = 9.4$, find PX .
- If Y is a point on $\odot T$ and $PR = 12.8$, find PY .

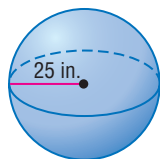


- GRILLS** A hemispherical barbecue grill has two racks, one for the food and one for the charcoal. The food rack is a great circle of the grill and has a radius of 11 inches. The charcoal rack is 5 inches below the food rack. Find the difference in the areas of the two racks.

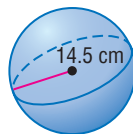


Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

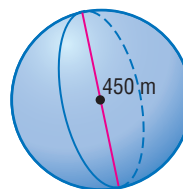
17.



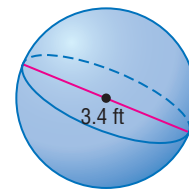
18.



19.



20.



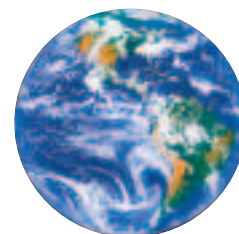
21. Hemisphere: The circumference of a great circle is 40.8 inches.
22. Sphere: The circumference of a great circle is 30.2 feet.
23. Sphere: The area of a great circle is 814.3 square meters.
24. Hemisphere: The area of a great circle is 227.0 square kilometers.

Determine whether each statement is *true* or *false*. If false, give a counterexample.

25. The radii of a sphere are congruent to the radius of its great circle.
26. In a sphere, two different great circles intersect in only one point.
27. Two spheres with congruent radii can intersect in a circle.
28. A sphere's longest chord will pass through the center of the circle.
29. Two spheres can intersect in one point.

EARTH For Exercises 30–32, use the following information.

The diameter of Earth is 7899.83 miles from the North Pole to the South Pole and 7926.41 miles from opposite points at the equator.



30. Find the surface area of Earth using each measure.
31. If the atmosphere of Earth extends to about 100 miles above the surface, find the surface area of the atmosphere surrounding Earth. Use the mean of the two diameters.
32. About 75% of Earth's surface is covered by water. Find the surface area of water on Earth, using the mean of the two diameters.

33. **IGLOOS** Use the information at the left to find the surface area of the living area if the diameter is 13 feet.
34. Find the ratio of the surface area of two spheres if the radius of one is twice the radius of the second sphere.
35. Find the ratio of the radii of two spheres if the surface area of one is one half the surface area of the other.
36. Find the ratio of the surface areas of two spheres if the radius of one is three times the radius of the other.



More About . . .

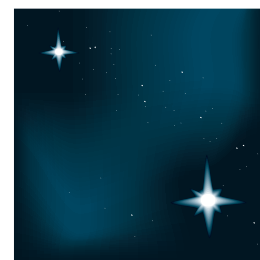
Igloos

Igloos are made of hard-packed snow cut into blocks. The blocks are put together in a spiral that grows smaller at the top to form a hemispherical dome.

Source: www.infoplease.com

ASTRONOMY For Exercises 37 and 38, use the following information.

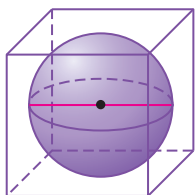
In 2002, NASA's Chandra X-Ray Observatory found two unusual neutron stars. These two stars are smaller than previously found neutron stars, but they have the mass of a larger neutron star, causing astronomers to think this star may not be made of neutrons, but a different form of matter.



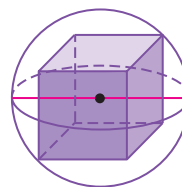
37. Neutron stars have diameters from 12 to 20 miles in size. Find the range of the surface area.
38. One of the new stars has a diameter of 7 miles. Find the surface area of this star.



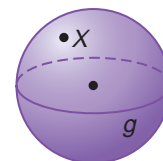
39. A sphere is inscribed in a cube. Describe how the radius of the sphere is related to the dimensions of the cube.



40. A sphere is circumscribed about a cube. Find the length of the radius of the sphere in terms of the dimensions of the cube.



41. **CRITICAL THINKING** In spherical geometry, a plane is the surface of a sphere and a line is a great circle. How many lines exist that contain point X and do not intersect line g ?



42. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do manufacturers of sports equipment use the surface area of spheres?

Include the following in your answer:

- how to find the surface area of a sphere, and
- other examples of sports that use spheres.

Standardized Test Practice

A B C D

43. A rectangular solid that is 4 inches long, 5 inches high, and 7 inches wide is inscribed in a sphere. What is the radius of this sphere?

- (A) 4.74 in. (B) 5.66 in. (C) 7.29 in. (D) 9.49 in.

44. **ALGEBRA** Solve $\sqrt{x^2 + 7} - 2 = x - 1$.

- (A) -3 (B) $\frac{1}{3}$ (C) 3 (D) no solution

Maintain Your Skills

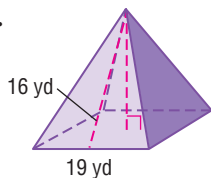
Mixed Review

Find the surface area of each cone. Round to the nearest tenth. (Lesson 12-6)

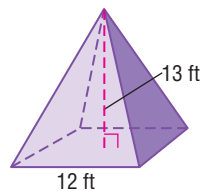
45. $h = 13$ inches, $\ell = 19$ inches 46. $r = 7$ meters, $h = 10$ meters
 47. $r = 4.2$ cm, $\ell = 15.1$ cm 48. $d = 11.2$ ft, $h = 7.4$ ft

Find the surface area of each regular pyramid. Round to the nearest tenth if necessary. (Lesson 12-5)

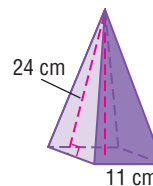
49.



50.



51.



52. **RECREATION** Find the area of fabric needed to cover one side of a frisbee with a diameter of 9 inches. Allow an additional 3 inches around the frisbee. (Lesson 11-3)

Write an equation for each circle. (Lesson 10-8)

53. a circle with center at $(-2, 7)$ and a radius with endpoint at $(3, 2)$
 54. a diameter with endpoints at $(6, -8)$ and $(2, 5)$



Geometry Activity

A Follow-Up of Lesson 12-7

Locus and Spheres

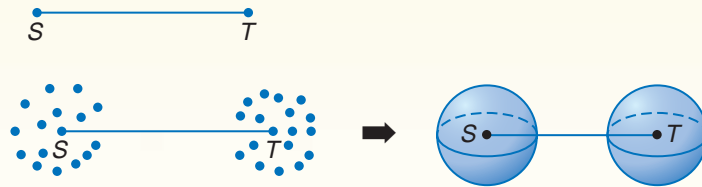
Spheres are defined in terms of a locus of points in space. The definition of a sphere is the set of all points that are a given distance from a given point.

Activity 1

Find the locus of points a given distance from the endpoints of a segment.

Collect the Data

- Draw a given line segment with endpoints S and T .
- Create a set of points that are equidistant from S and a set of points that are equidistant from T .



Analyze

1. Draw a figure and describe the locus of points in space that are 5 units from each endpoint of a given segment that is 25 units long.
2. Are the two spheres congruent?
3. What are the radii and diameters of each sphere?
4. Find the distance between the two spheres.

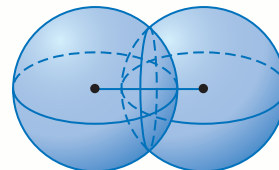
Activity 2

Investigate spheres that intersect.

Find the locus of all points that are equidistant from the centers of two intersecting spheres with the same radius.

Collect the Data

- Draw a line segment. 
- Draw congruent overlapping spheres, with the centers at the endpoints of the given line segment.



Analyze

5. What is the shape of the intersection of the spheres?
6. Can this be described as a locus of points in space or on a plane? Explain.
7. Describe the intersection as a locus.
8. **MINING** What is the locus of points that describes how particles will disperse in an explosion at ground level if the expected distance a particle could travel is 300 feet?



Vocabulary and Concept Check

axis (p. 655)	great circle (p. 671)	oblique prism (p. 649)	regular prism (p. 637)
bases (p. 637)	hemisphere (p. 672)	orthogonal drawing (p. 636)	regular pyramid (p. 660)
circular cone (p. 666)	lateral area (p. 649)	perspective view (p. 636)	right cone (p. 666)
cone (p. 638)	lateral edges (p. 649)	Platonic Solids (p. 637)	right cylinder (p. 655)
corner view (p. 636)	lateral faces (p. 649)	polyhedron (p. 637)	right prism (p. 649)
cross section (p. 639)	net (p. 644)	prism (p. 637)	slant height (p. 660)
cylinder (p. 638)	oblique cone (p. 666)	pyramid (p. 637)	sphere (p. 638)
edges (p. 637)	oblique cylinder (p. 655)	regular polyhedron (p. 637)	surface area (p. 644)
face (p. 637)			

A complete list of postulates and theorems can be found on pages R1–R8.

Exercises Match each expression with the correct formula.

- lateral area of a prism
- surface area of a prism
- lateral area of a cylinder
- surface area of a cylinder
- lateral area of a regular pyramid
- surface area of a regular pyramid
- lateral area of a cone
- surface area of a cone
- surface area of a sphere
- surface area of a cube

a. $L = \frac{1}{2}P\ell$	f. $T = 6s^2$
b. $L = 2\pi rh$	g. $T = \pi r\ell + \pi r^2$
c. $T = 4\pi r^2$	h. $T = 2\pi rh + 2\pi r^2$
d. $L = Ph$	i. $T = Ph + 2B$
e. $L = \pi r\ell$	j. $T = \frac{1}{2}P\ell + B$

Lesson-by-Lesson Review

12-1 Three-Dimensional Figures

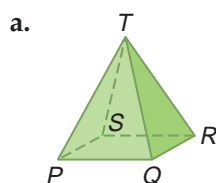
See pages
636–642.

Concept Summary

- A solid can be determined from its orthogonal drawing.
- Solids can be classified by bases, faces, edges, and vertices.

Examples

Identify each solid. Name the bases, faces, edges, and vertices.



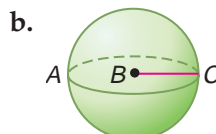
The base is a rectangle, and all of the lateral faces intersect at point T , so this solid is a rectangular pyramid.

Base: $\square PQRS$

Faces: $\triangle TPQ, \triangle TQR, \triangle TRS, \triangle TSP$

Edges: $\overline{PQ}, \overline{QR}, \overline{RS}, \overline{PS}, \overline{PT}, \overline{QT}, \overline{RT}, \overline{ST}$

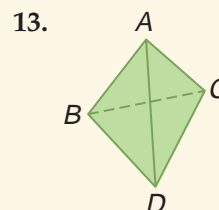
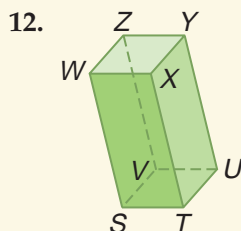
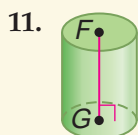
Vertices: P, Q, R, S, T



This solid has no bases, faces, or edges. It is a sphere.

Exercises Identify each solid. Name the bases, faces, edges, and vertices.

See Example 2 on page 638.



12-2 Nets and Surface Area

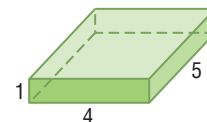
See pages 643–648.

Concept Summary

- Every three-dimensional solid can be represented by one or more two-dimensional nets.
- The area of the net of a solid is the same as the surface area of the solid.

Examples

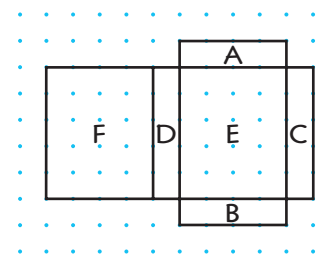
Draw a net and find the surface area for the right rectangular prism shown.



Use rectangular dot paper to draw a net. Since each face is a rectangle, opposite sides have the same measure.

To find the surface area of the prism, add the areas of the six rectangles.

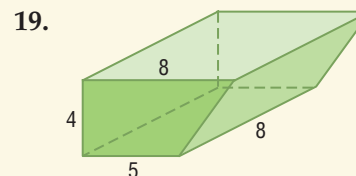
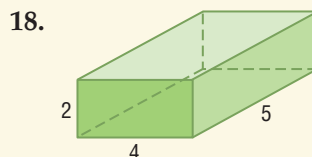
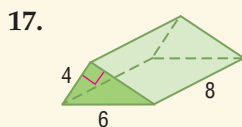
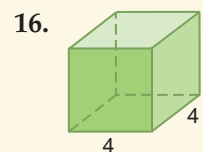
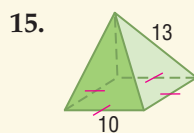
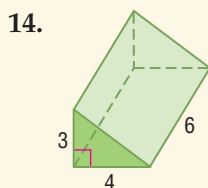
$$\begin{aligned} \text{Surface area} &= A + B + C + D + E + F \\ &= 4 \cdot 1 + 4 \cdot 1 + 5 \cdot 1 + 5 \cdot 1 + 4 \cdot 5 + 4 \cdot 5 \\ &= 4 + 4 + 5 + 5 + 20 + 20 \\ &= 58 \end{aligned}$$



The surface area is 58 square units.

Exercises For each solid, draw a net and find the surface area.

See Example 3 on page 645.



12-3 Surface Areas of Prisms

See pages
649–654.

Concept Summary

- The lateral faces of a prism are the faces that are not bases of the prism.
- The lateral surface area of a right prism is the perimeter of a base of the prism times the height of the prism.

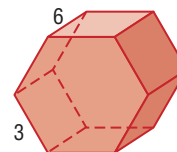
Example

Find the lateral area of the regular hexagonal prism.

The bases are regular hexagons. So the perimeter of one base is $6(3)$ or 18. Substitute this value into the formula.

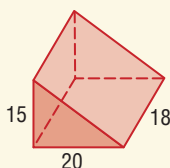
$$\begin{aligned} L &= Ph && \text{Lateral area of a prism} \\ &= (18)(6) && P = 18, h = 6 \\ &= 108 && \text{Multiply.} \end{aligned}$$

The lateral area is 108 square units.

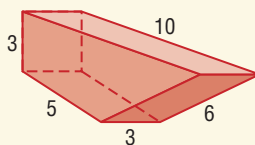


Exercises Find the lateral area of each prism. See Example 1 on page 650.

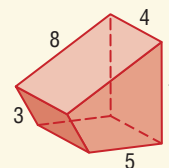
20.



21.



22.



12-4 Surface Areas of Cylinders

See pages
655–659.

Concept Summary

- The lateral surface area of a cylinder is 2π multiplied by the product of the radius of a base of the cylinder and the height of the cylinder.
- The surface area of a cylinder is the lateral surface area plus the area of both circular bases.

Example

Find the surface area of a cylinder with a radius of 38 centimeters and a height of 123 centimeters.

$$\begin{aligned} T &= 2\pi rh + 2\pi r^2 && \text{Surface area of a cylinder} \\ &= 2\pi(38)(123) + 2\pi(38)^2 && r = 38, h = 123 \\ &\approx 38,440.5 && \text{Use a calculator.} \end{aligned}$$

The surface area of the cylinder is approximately 38,440.5 square centimeters.

Exercises Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth. See Example 2 on page 656.

23. $d = 4$ in., $h = 12$ in.

24. $r = 6$ ft, $h = 8$ ft

25. $r = 4$ mm, $h = 58$ mm

26. $d = 4$ km, $h = 8$ km

12-5 Surface Areas of Pyramids

See pages
660–665.

Concept Summary

- The slant height ℓ of a regular pyramid is the length of an altitude of a lateral face.
- The lateral area of a pyramid is $\frac{1}{2}P\ell$, where ℓ is the slant height of the pyramid and P is the perimeter of the base of the pyramid.

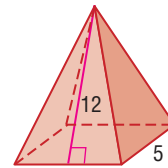
Example

Find the surface area of the regular pyramid.

The perimeter of the base is $4(5)$ or 20 units, and the area of the base is 5^2 or 25 square units. Substitute these values into the formula for the surface area of a pyramid.

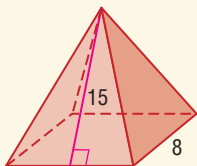
$$\begin{aligned} T &= \frac{1}{2}P\ell + B && \text{Surface area of a regular pyramid} \\ &= \frac{1}{2}(20)(12) + 25 && P = 20, \ell = 12, B = 25 \\ &= 145 && \text{Simplify.} \end{aligned}$$

The surface area is 145 square units.

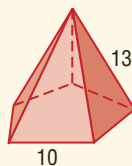


Exercises Find the surface area of each regular pyramid. Round to the nearest tenth if necessary. See Example 2 on pages 661 and 662.

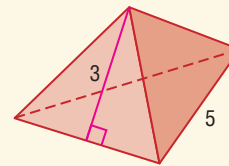
27.



28.



29.



12-6 Surface Areas of Cones

See pages
666–670.

Concept Summary

- A cone is a solid with a circular base and a single vertex.
- The lateral area of a right cone is $\pi r\ell$, where ℓ is the slant height of the cone and r is the radius of the circular base.

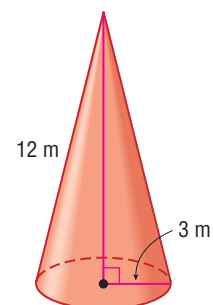
Example

Find the surface area of the cone.

Substitute the known values into the formula for the surface area of a right cylinder.

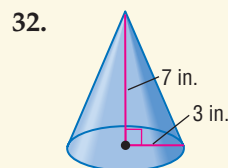
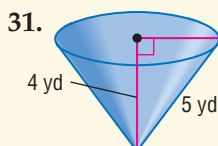
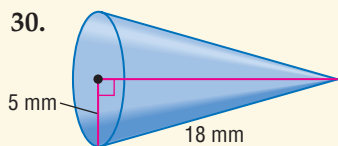
$$\begin{aligned} T &= \pi r\ell + \pi r^2 && \text{Surface area of a cone} \\ T &= \pi(3)(12) + \pi(3)^2 && r = 3, \ell = 12 \\ T &\approx 141.4 && \text{Use a calculator.} \end{aligned}$$

The surface area is approximately 141.4 square meters.



Exercises Find the surface area of each cone. Round to the nearest tenth.

See Example 2 on page 667.



12-7 Surface Areas of Spheres

See pages 671–676.

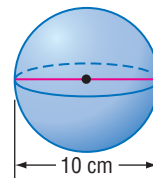
Concept Summary

- The set of all points in space a given distance from one point is a sphere.
- The surface area of a sphere is $4\pi r^2$, where r is the radius of the sphere.

Examples

- a. Find the surface area of a sphere with a diameter of 10 centimeters.

$$\begin{aligned} T &= 4\pi r^2 && \text{Surface area of a sphere} \\ &= 4\pi(5)^2 && r = 5 \\ &\approx 314.2 && \text{Use a calculator.} \end{aligned}$$



The surface area is approximately 314.2 square centimeters.

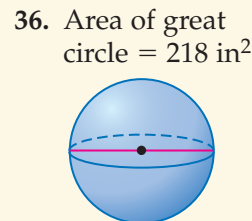
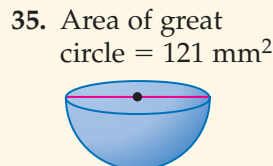
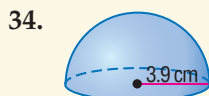
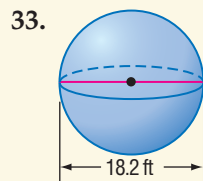
- b. Find the surface area of a hemisphere with radius 6.3 inches.

To find the surface area of a hemisphere, add the area of the great circle to half of the surface area of the sphere.

$$\begin{aligned} \text{surface area} &= \frac{1}{2}(4\pi r^2) + \pi r^2 && \text{Surface area of a hemisphere} \\ &= \frac{1}{2}[4\pi(6.3)^2] + \pi(6.3)^2 && r = 6.3 \\ &\approx 374.1 && \text{Use a calculator.} \end{aligned}$$

The surface area is approximately 374.1 square inches.

Exercises Find the surface area of each sphere or hemisphere. Round to the nearest tenth if necessary. See Example 2 on page 673.



37. a hemisphere with radius 16 ft
38. a sphere with diameter 5 m
39. a sphere that has a great circle with an area of 220 ft^2
40. a hemisphere that has a great circle with an area of 30 cm^2

Vocabulary and Concepts

Match each expression to the correct formula.

- surface area of a prism
- surface area of a cylinder
- surface area of a regular pyramid

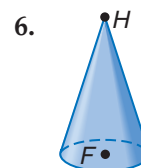
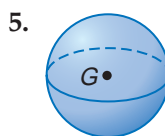
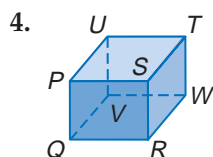
$$\text{a. } T = 2\pi rh + 2\pi r^2$$

$$\text{b. } T = \frac{1}{2}P\ell + B$$

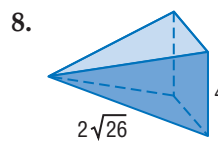
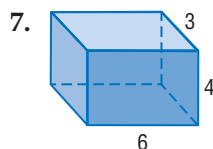
$$\text{c. } T = Ph + 2B$$

Skills and Applications

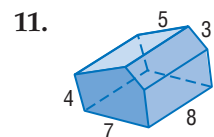
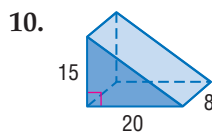
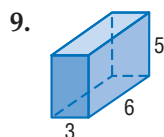
Identify each solid. Name the bases, faces, edges, and vertices.



For each solid, draw a net and find the surface area.



Find the lateral area of each prism.



Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

12. $r = 8$ ft, $h = 22$ ft

13. $r = 3$ mm, $h = 2$ mm

14. $r = 78$ m, $h = 100$ m

The figure at the right is a composite solid of a tetrahedron and a triangular prism. Find each measure in the solid. Round to the nearest tenth if necessary.

15. height

16. lateral area

17. surface area



Find the surface area of each cone. Round to the nearest tenth.

18. $h = 24$, $r = 7$

19. $h = 3$ m, $\ell = 4$ m

20. $r = 7$, $\ell = 12$

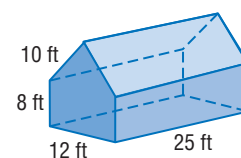
Find the surface area of each sphere. Round to the nearest tenth if necessary.

21. $r = 15$ in.

22. $d = 14$ m

23. The area of a great circle of the sphere is 116 square feet.

24. **GARDENING** The surface of a greenhouse is covered with plastic or glass. Find the amount of plastic needed to cover the greenhouse shown.



25. **STANDARDIZED TEST PRACTICE** A cube has a surface area of 150 square centimeters. What is the length of each edge?

(A) 25 cm

(B) 15 cm

(C) 12.5 cm

(D) 5 cm

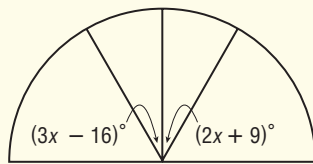


Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. A decorative strip of wood is used to bisect the center triangular window pane. What is the measure of each angle formed when the pane is bisected? (Lesson 1-5)

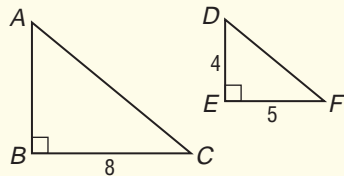
- (A) 12°
- (B) 25°
- (C) 50°
- (D) 59°



2. The coordinates of the endpoints of a segment are (0, 1) and (6, 9). A congruent segment has one endpoint at (10, 6). What are the coordinates of the other endpoint? (Lesson 4-3)

- (A) (10, 15)
- (B) (14, 14)
- (C) (16, 15)
- (D) (16, 14)

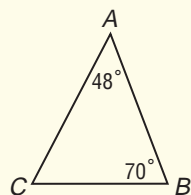
3. Which of the following additional information is needed to prove that $\triangle ABC$ is similar to $\triangle DEF$? (Lesson 6-3)



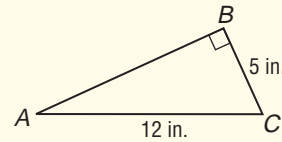
- (A) $\triangle ABC$ and $\triangle DEF$ are right triangles.
- (B) The length of \overline{AB} is proportional to the length of \overline{DE} .
- (C) Two pairs of corresponding angles are congruent.
- (D) The length of \overline{BC} is twice the length of \overline{DE} .

4. Which of the following lists the sides of $\triangle ABC$ in order from longest to shortest? (Lesson 5-3)

- (A) $\overline{BC}, \overline{AB}, \overline{AC}$
- (B) $\overline{AC}, \overline{AB}, \overline{BC}$
- (C) $\overline{AC}, \overline{BC}, \overline{AB}$
- (D) $\overline{BC}, \overline{AC}, \overline{AB}$



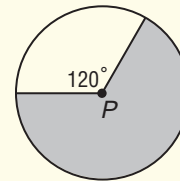
5. What is the length of \overline{AB} ? (Lesson 7-2)



- (A) 8.9 in.
- (B) 10.9 in.
- (C) 12 in.
- (D) 13 in.

6. The diameter of circle P is 18 inches. What is the area of the shaded region? (Lesson 11-5)

- (A) 27π
- (B) 54π
- (C) 81π
- (D) 216π



7. Which statement is false? (Lesson 12-1)

- (A) A pyramid is a polyhedron.
- (B) The bases of a cylinder are in parallel planes.
- (C) All of the Platonic Solids are regular prisms.
- (D) A cone has a vertex.

8. Shelly bought a triangular prism at the science museum. The bases of the prism are equilateral triangles with side lengths of 2 centimeters. The height of the prism is 4 centimeters. What is the surface area of Shelly's prism to the nearest square centimeter? (Lesson 12-3)

- (A) 16 cm^2
- (B) 27 cm^2
- (C) 28 cm^2
- (D) 31 cm^2

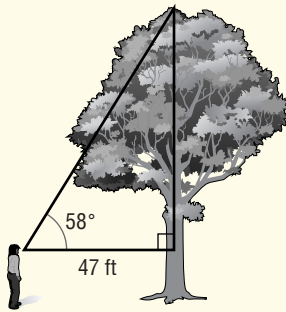
9. A spherical weather balloon has a diameter of 4 feet. What is the surface area of the balloon to the nearest square foot? (Lesson 12-7)

- (A) 50 ft^2
- (B) 25 ft^2
- (C) 16 ft^2
- (D) 13 ft^2

Part 2 Short Response/Grid In

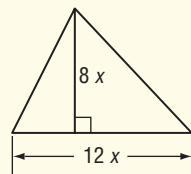
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. Samantha wants to estimate the height of the tree.

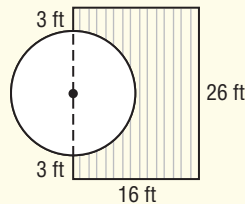


If Samantha is 5 feet tall, what is the height of the tree to the nearest foot? (Lesson 7-5)

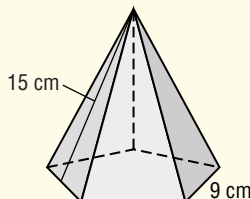
11. If the base and height of the triangle are decreased by $2x$ units, what is the area of the resulting triangle? (Lesson 11-2)



12. Mr. Jiliana built a wooden deck around half of his circular swimming pool. He needs to know the area of the deck so he can buy cans of stain. What is the area, to the nearest square foot, of the deck? (Lesson 11-4)



13. What is the surface area of this regular pentagonal pyramid to the nearest tenth of a square centimeter? (Lesson 12-5)



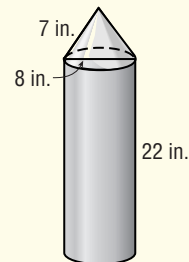
Test-Taking Tip

Question 2 Sometimes backsolving is the best technique to use to solve a multiple-choice test question. Check each of the given choices until you find the correct coordinates.

Part 3 Open Ended

Record your answers on a sheet of paper. Show your work.

14. A cylindrical pole is being used to support a large tent. The diameter of the base is 18 inches, and the height is 15 feet. (Lessons 12-2 and 12-4)
- Draw a net of the cylinder and label the dimensions.
 - What is the lateral area of the pole to the nearest square foot?
 - What is the surface area of the pole to the nearest square foot?
15. Aliya is constructing a model of a rocket. She uses a right cylinder for the base and a right cone for the top. (Lessons 12-4 and 12-6)



- What is the surface area of the cone to the nearest square inch?
- What is the surface area of the cylinder to the nearest square inch?
- What is the surface area of the rocket, once it is assembled? Round to the nearest square inch.